#CRESTInnovator

CREST Mathematics Olympiad (CMO) Worksheet for Class 9

Topic

Polynomials

Worksheet on Polynomials

1. **What must be added to $p(x) = 2x^3 - 3x^2 + 5x - 16$ to obtain a polynomial which is exactly divisible by $g(x) = (x - 2)$?**

   a. -2
   b. 2
   c. 3
   d. 4

2. **If $p(x) = ax^3 + bx^2 - 4x + 3$ has $g(x) = x - 3$ as a factor and leaves a remainder of 33 when divided by $h(x) = x + 3$, what are the values of a and b?**

   a. $a = 1/6$ and $b = 3/2$
   b. $a = -1/6$ and $b = 3/2$
   c. $a = 1/6$ and $b = -3/2$
   d. $a = -1/6$ and $b = -3/2$

3. **What is the remainder left when $p(x) = x^4 - 4x^3 - x^2 + 16x - 12$ is divided by $g(x) = x^2 - 5x + 6$?**

   a. 0
   b. 1
   c. 2
   d. 3

4. **If the polynomials $p(x) = 3x^3 + bx^2 + 5x - 8$ and $q(x) = 2x^3 + 3x^2 - 3x + b$ leave the same remainder when divided by $g(x) = (x - 3)$, what is the value of b and the remainder?**

   a. b = 2, remainder = 70
   b. b = 2, remainder = −70
   c. b = −2, remainder = 70
   d. b = −2, remainder = −70

5. **An expression Q when simplified becomes $p(x) \times B$, where B is one of the factors of Q and $p(x) = 2x^3 - 3x^2 + 2x - 33$. Another expression Y when simplified becomes $g(x) \times C$, where C is one of the factors of Y and $g(x) = x - 3$. If Q is the number of students in a school and Y is the number of chocolates to be distributed among them, then what is the chance that every student gets an equal number of chocolates?**

   a. Every student gets an equal number of chocolates.
   b. Every student doesn’t get an equal number of chocolates.
   c. Every second student gets an equal number of chocolates.
   d. Every second student doesn’t get an equal number of chocolates.

Answer Key

1. b - 2

Explanation: Here, p(x) is not completely divisible by g(x).
To make it completely divisible we need to add a constant.
Let the required number to be added be k.
Now,
$p(x) = 2x^3 - 3x^2 + 5x - 16 + k$
$g(x) = (x - 2)$
By factor theorem, p(x) will be divisible by g(x) when p(2) = 0.
$p(2) = 0$
$\Rightarrow 2 \times 2^3 - 3 \times 2^2 + 5 \times 2 - 16 + k = 0$
$\Rightarrow 16 - 12 + 10 - 16 + k = 0$
$\Rightarrow -2 + k = 0$
$\therefore k = 2$
2 must be added to p(x) to obtain a polynomial that is exactly divisible by g(x).

2. b - a = −1/6 and b = 3/2

Explanation: Given: $p(x) = ax^3 + bx^2 - 4x + 3$
According to the Factor theorem, we write g(x) = 0 to get the value of x for which p(x) must be zero as g(x) is one of the factors of p(x).
$g(x) = 0$
$\Rightarrow x - 3 = 0$
$\therefore x = 3$
If (x − 3) is a factor of p(x), then p(3) = 0.
$\therefore p(3) = 0$
$\Rightarrow a \times 3^3 + b \times 3^2 - 4 \times 3 + 3 = 0$
$\Rightarrow 27a + 9b - 12 + 3 = 0$
$\Rightarrow 27a + 9b = 9$
$\therefore 3a + b = 1$ ……(i)
As per the Remainder theorem, when p(x) is divided by h(x) = (x + 3), then the remainder is p(−3) which is 33(given).
$ax^3 + bx^2 - 4x + 3$
$\therefore p(-3) = 33$
$\Rightarrow a \times (-3)^3 + b \times (-3)^2 - 4 \times (-3) + 3 = 33$
$\Rightarrow -27a + 9b + 12 + 3 = 33$
$\Rightarrow -27a + 9b = 18$
$\therefore -3a + b = 2$ ……..(ii)
Adding (i) and (ii), we get:
$3a + b - 3a + b = 1 + 2$
$\Rightarrow 2b = 3$
$\therefore b = 3/2$
Putting the value of b in (i), we get:
$3a + 3/2 = 1$
$\Rightarrow 3a = 1 - 3/2$
$\Rightarrow 3a = -1/2$
$\therefore a = -1/6$

3. a - 0

Explanation: $g(x) = x^2 - 5x + 6$
$= x^2 - 3x - 2x + 6$
$= x(x - 3) - 2(x - 3)$
$= (x - 3)(x - 2)$
p(x) will be exactly divisible by g(x) only when it is exactly divisible by (x − 3) as well as (x − 2).
If g(x) doesn't divide p(x) completely, then it leaves the remainder.
We write g(x) = 0 to get the value of x for which p(x) must be zero.
$(x - 3) = 0$
$\therefore x = 3$
$(x - 2) = 0$
$\therefore x = 2$
By the factor theorem, g(x) will be a factor of p(x), if p(3) = 0 and p(2) = 0.
$\therefore p(3) = 3^4 - 4 \times 3^3 - 3^2 + 16 \times 3 - 12$
$= 81 - 108 - 9 + 48 - 12$
$= 129 - 129$
$= 0$
$\therefore p(2) = 2^4 - 4 \times 2^3 - 2^2 + 16 \times 2 - 12$
$= 16 - 32 - 4 + 32 - 12$
$= 12 - 12$
$= 0$
As p(3) = 0 and p(2) = 0, p(x) is exactly divisible by each of (x − 3) and (x − 2), respectively.
Hence, p(x) is exactly divisible by g(x) leaving the remainder zero.

4. c - b = −2, remainder = 70

Explanation: According to the Remainder theorem,
If p(x) is divided by (x − 3), then the remainder is p(3).
If $p(x) = 3x^3 + bx^2 + 5x - 8$, then
$\therefore p(3) = 3 \times 3^3 + b \times 3^2 + 5 \times 3 - 8$
$= 81 + 9b + 15 - 8$
$= 88 + 9b$
If q(x) is divided by (x − 3) remainder is q(3).
If $q(x) = 2x^3 + 3x^2 - 3x + b$, then
$\therefore q(x) = 2 \times 3^3 + 3 \times 3^2 - 3 \times 3 + b$
$= 54 + 27 - 9 + b$
$= 72 + b$
As the remainder is the same in both cases:
$\Rightarrow 88 + 9b = 72 + b$
$\Rightarrow 9b - b = 72 - 88$
$\Rightarrow 8b = -16$
$\therefore b = -2$
As the remainder is the same:
$\therefore$ Remainder = 72 + b
$= 72 - 2$
$= 70$

5. a - Every student gets an equal number of chocolates.

Explanation: Every student will get an equal number of chocolates only when Q is a multiple of Y because then only Y will divide Q with a zero remainder, thus giving every student an equal number of chocolates.
The given factors in expressions Q and Y are p(x) and g(x). We will divide p(x) by g(x) and if the remainder comes to zero then every student has an equal number of chocolates.
By the remainder theorem, we know that when $p(x) = 2x^3 - 3x^2 + 2x - 33$ is divided by $g(x) = x - 3$ then the remainder is p(3).
Thus, $p(3) = 2 \times 3^3 - 3 \times 3^2 + 2 \times 3 - 33$
$= 54 - 27 + 6 - 33 = 0$
As the remainder comes to zero, p(x) is a multiple of g(x), and this means that g(x) completely divides p(x), thus giving every student an equal number of chocolates.

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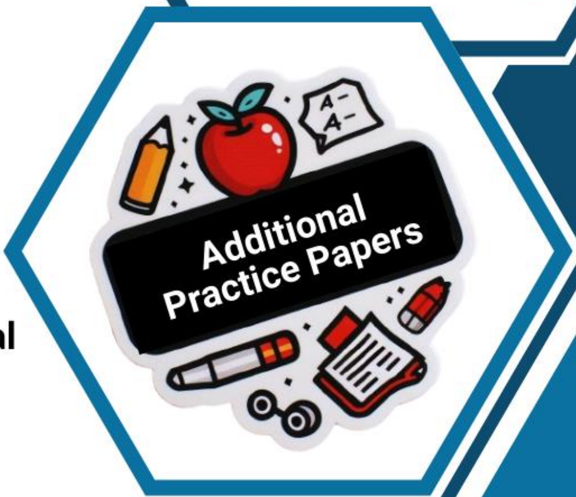

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