CREST Olympiads

#CRESTInnovator

CREST Mathematics Olympiad (CMO)

Worksheet *for*

Class 10

Topic

Trigonometry

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Worksheet on Trigonometry

1. **If sin A : cos A = 3 : 9, then what is the value of the given trigonometric expression?**

$$\frac{5\cos A + 6\sin A}{3\cos A - 2\sin A}$$

a. 1
b. 2
c. 0
d. 3

2. **If cot A + tan A = p and sec A − cos A = q, then what is the value of the given expression?**

$$(p^2q)^{\frac{2}{3}} - (pq^2)^{\frac{2}{3}}$$

a. 0
b. 1
c. 2
d. -2

3. **If sin A = cos A/√3, then what is the value of $\sin^2 A + \sec^2 A + \sin A \sec A + 2 \operatorname{cosec} A$?**

a. $\frac{67 + 4\sqrt{3}}{12}$

b. $\frac{67 + \sqrt{3}}{12}$

c. $\frac{65 + 4\sqrt{3}}{12}$

d. $\frac{65 + \sqrt{3}}{12}$

4. **A woman on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. 18 minutes later, the angle of depression of the boat was found to be 60°. What is the total time taken by the boat to reach the shore?**

   a. 9 minutes
   b. 25 minutes
   c. 27 minutes
   d. 28 minutes

5. **Simplify the following trigonometric expression:**

$$\frac{(1 + \cos\theta^2) + \sin^2\theta}{(\text{cosec}^2\theta - 1)\sin^2\theta}$$

   a. 2cos θ (1 + sec θ)
   b. sec θ (1 + sec θ)
   c. cos θ (1 + sec θ)
   d. 2sec θ (1 + sec θ)

Answer Key

1. d - 3

Explanation:

Given:- $\sin A : \cos A = 3 : 9$

$\rightarrow \dfrac{\sin A}{\cos A} = \dfrac{3}{9}$

$\rightarrow \tan A = \dfrac{1}{3}$

We know that $\tan A = \dfrac{\text{Perpendicular}}{\text{Base}}$

$\rightarrow$ Perpendicular = 1 and Base = 3

In a triangle,

$(\text{Hypotenuse}) = (\text{Perpendicular})^2 + (\text{Base})^2$

$(\text{Hypotenuse}) = (1)^2 + (3)^2$

$= 1 + 9$

$= 10$

Hypotenuse = $\sqrt{10}$

Now, we know that:

$\sin A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{1}{\sqrt{10}}$

$\cos A = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{3}{\sqrt{10}}$

$$\rightarrow \frac{5\cos A + 6\sin A}{3\cos A - 2\sin A} = \frac{5\left(\frac{3}{\sqrt{10}}\right) + 6\left(\frac{1}{\sqrt{10}}\right)}{3\left(\frac{3}{\sqrt{10}}\right) - 2\left(\frac{1}{\sqrt{10}}\right)}$$

$$= \frac{15 + 6}{9 - 2}$$

$$= \frac{21}{7}$$

$$= 3$$

2. b - 1

Explanation:

We are given $\cot A + \tan A = p$

$\rightarrow \frac{1}{\tan A} + \tan A = p$

$\rightarrow \frac{1 + \tan^2 A}{\tan A} = p$

We know $1 + \tan^2 A = \sec^2 A$

$\rightarrow \frac{\sec A}{\tan A} = p$

$\rightarrow p = \frac{1}{\cos^2 A} \times \frac{\cos A}{\sin A}$

$\rightarrow p = \frac{1}{\sin A \cos A}$

Now $q = \sec A - \cos A$

$\rightarrow q = \frac{1}{\cos A} - \cos A$

$= \frac{1 - \cos^2 A}{\cos A}$

We know that $\sin^2 A + \cos^2 A = 1$

$q = \frac{\sin^2 A}{\cos A}$

Now,

$$(p^2 q)^{\frac{2}{3}} - (pq^2)^{\frac{2}{3}} = \left[\left(\frac{1}{\sin A \cos A}\right)^2 \times \frac{\sin^2 A}{\cos A}\right]^{\frac{2}{3}} - \left[\frac{1}{\sin A \cos A} \times \left(\frac{\sin^2 A}{\cos A}\right)^2\right]^{\frac{2}{3}}$$

$$= \left[\frac{1}{\sin^2 A \cos^2 A} \times \frac{\sin^2 A}{\cos A}\right]^{\frac{2}{3}} - \left[\frac{1}{\sin A \cos A} \times \frac{\sin^4 A}{\cos^2 A}\right]^{\frac{2}{3}}$$

$$= \left[\frac{1}{\cos^3 A}\right]^{\frac{2}{3}} - \left[\frac{\sin^3 A}{\cos^3 A}\right]^{\frac{2}{3}}$$

$$= (\sec^3 A)^{\frac{2}{3}} - (\tan^3 A)^{\frac{2}{3}}$$

$$= \sec^2 A - \tan^2 A$$

$= 1$ (Using identity $1 + \tan^2 A = \sec^2 A$)

$\therefore (p^2 q)^{\frac{2}{3}} - (pq^2)^{\frac{2}{3}} = 1$

3. a - $\boxed{\dfrac{65 + 4\sqrt{3}}{12}}$

Explanation:

We are given $\cot A = \cos A / \sqrt{3}$

$\rightarrow \dfrac{\sin A}{\cos A} = \dfrac{1}{\sqrt{3}}$

$\rightarrow \tan A = \dfrac{1}{\sqrt{3}}$

We know that $\tan 30° = \dfrac{1}{\sqrt{3}}$

$\rightarrow A = 30°$

Thus, $\sin 30° = \dfrac{1}{2}$

$\sec 30° = \dfrac{2}{\sqrt{3}}$

$\operatorname{cosec} 30° = 2$

$\rightarrow \sin^2 A + \sec^2 A + \sin A \sec A + 2 \operatorname{cosec} A$

$= (\sin A)^2 + (\sec A)^2 + \sin A \sec A + 2 \operatorname{cosec} A$

For $A = 30°$

$\rightarrow \sin^2 30° + \sec^2 30° + \sin 30° \sec 30° + 2 \operatorname{cosec} 30°$

$= (\sin 30°)^2 + (\sec 30°)^2 + (\sin 30°) \times (\sec 30°) + 2 \operatorname{cosec} 30°$

$= \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{2}{\sqrt{3}}\right)^2 + \left(\dfrac{1}{2}\right)\left(\dfrac{2}{\sqrt{3}}\right) + 2(2)$

$= \dfrac{1}{4} + \dfrac{4}{3} + \dfrac{1}{\sqrt{3}} + 4$

$= \dfrac{3 + 16 + 48}{12} + \dfrac{1}{\sqrt{3}}$

$= \dfrac{67}{12} + \dfrac{1}{\sqrt{3}}$

$= \dfrac{67}{12} + \dfrac{1}{\sqrt{3}}$

$= \dfrac{67}{12} + \dfrac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

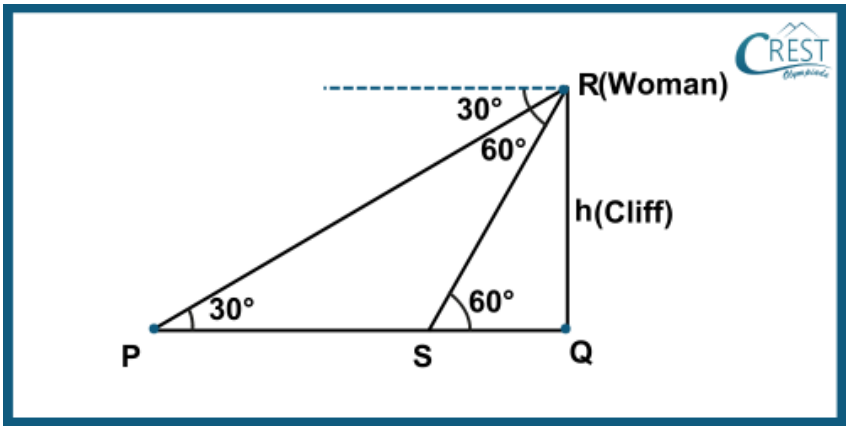
$= \dfrac{67}{12} + \dfrac{\sqrt{3}}{3}$

$= \dfrac{67 + 4\sqrt{3}}{12}$ $\rightarrow \sin^2 A + \sec^2 A + \sin A \sec A + 2 \operatorname{cosec} A = \dfrac{67 + 4\sqrt{3}}{12}$

4. c - 27 minutes

Explanation: The two positions of the boat at the two instants are P and S and let the speed of the boat be x m/min, h be the height of the cliff and R be the location of the woman.

Thus, the figure is shown as:



The boat takes 18 minutes to reach point S from point P.

We know that: **Distance = Speed × Time**

Thus, the distance covered by the boat from point P to point S is:
PS = x × 18
PS = 18x

Let the boat take t time to reach the shore, then the distance covered from point S to point Q is:
SQ = xt

Now, apply the trigonometric ratio in the triangle SQR,
$\tan 60° = {}^{\text{Perpendicular}}/_{\text{Base}}$

In the triangle, SQR, QR is the perpendicular and SQ is the base whose lengths are QR = h
SQ = xt
$\rightarrow \tan 60° = {}^{h}/_{xt}$
$\rightarrow \sqrt{3} = {}^{h}/_{xt}$
$\rightarrow$ **h = xt$\sqrt{3}$ … (1)**

Now, apply the trigonometric ratio in the triangle PQR,
$\tan 30° = {}^{\text{Perpendicular}}/_{\text{Base}}$

In the triangle PQR, QR is perpendicular and PQ is the base whose lengths are QR = h
PQ = PS + SQ = 18x + xt
$\rightarrow \tan 30° = h/(18x + xt)$
$\rightarrow 1\sqrt{3} = h/(18x + xt)$
$\rightarrow$ **$h = x(18 + t)/\sqrt{3}$ … (2)**

Compare the values h from the equation (1) and equation (2),
→ $xt\sqrt{3} = x(18 + t)/\sqrt{3}$
→ $xt\sqrt{3} \times \sqrt{3} = x(18 + t)$
→ $3xt = x(18 + t)$
→ $3t = 18 + t$
→ $3t - t = 18$
→ $2t = 18$
→ $t = {}^{18}/_{2}$
t = 9 mins

Thus, the boat takes 9 minutes to reach the shore from point S.

∴ Total time taken by the boat to reach shore Q from point P
= 18 + 9 = 27 minutes

5. d - 2sec θ (1 + sec θ)

Explanation:

$$\frac{(1 + \cos\theta^2) + \sin^2\theta}{(\operatorname{cosec}^2\theta - 1)\sin^2\theta} = \frac{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}{(\operatorname{cosec}^2\theta - 1)\sin^2\theta}$$

[Using $\sin^2 A + \cos^2 A = 1$ and $\operatorname{cosec}^2 A - \cot^2 A = 1$]

$$= \frac{1 + 2\cos + 1}{(\cot^2\theta)\sin^2\theta}$$

$$= \frac{2 + 2\cos\theta}{(\cot^2\theta)\sin^2\theta}$$

$$= \frac{2(1 + \cos\theta)}{\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\sin^2\theta} \qquad \left[\cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{2(1 + \cos\theta)}{\cos^2\theta}$$

$$= 2(\sec^2\theta + \sec\theta)$$

$$= 2\sec\theta(\sec\theta + 1)$$

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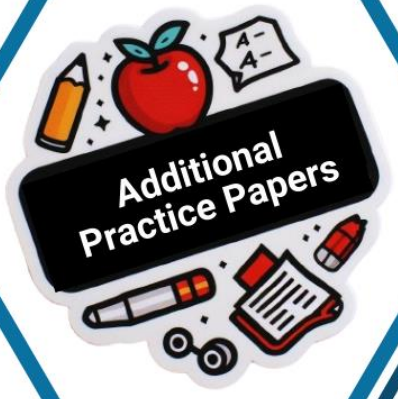

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