CREST Olympiads

#CRESTInnovator

CREST Mathematics Olympiad (CMO)

Worksheet for

Class 10

Topic

Triangles and Its Properties

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Worksheet on Triangles and Its Properties

1. **In △ABC, AB = 7 cm, BC = 3.5 cm, CA = 2√2 cm, ∠A = 75° and ∠B = 47° and in △PQR, PQ = 21 cm, QR = 10.5 cm, RP = 6√2 cm. What is the value of ∠P?**

   a. 75°
   b. 47°
   c. 58°
   d. 68°

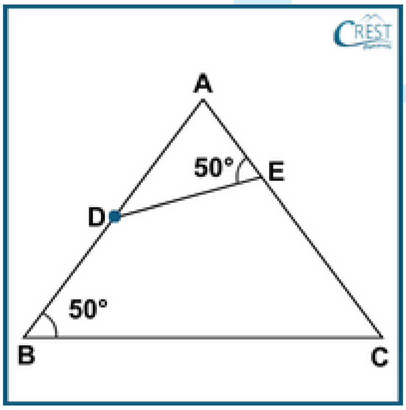
2. **Consider the following statements:**

   i. **If A and B are the points on the sides PQ and PR of △PQR such that PQ = 21 cm, PA = 6 cm, BR = 20 cm and PB = 8 cm, then AB || QR.**
   ii. **If in △ABC and △PQR, ∠A = 38°, ∠B = 57°, ∠P = 85° and ∠Q = 57°, then △BAC ~ △RQP.**

   Out of the following, which statement is TRUE?

   a. Only (i)
   b. Only (ii)
   c. Both (i) and (ii)
   d. Neither (i) nor (ii)

3. **Consider the figure given below:**



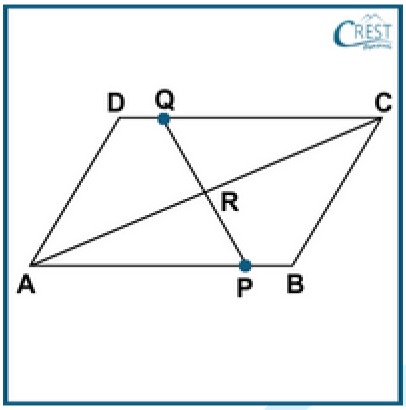
   ABC is a triangle with ∠B = 50° and AB = 12 cm. ADE is another triangle with ∠AED = 50°, AD = 7 cm and AE = 5 cm. What is the length of EC?

   a. 11. 5 cm
   b. 11.75 cm
   c. 11.6 cm
   d. 11.8 cm

4. **In a trapezium ABCD, AB || DC and DC = 3AB. EF drawn parallel to AB cuts AD at F and BC at E such that BE/EC = $\frac{4}{5}$. If diagonal DB intersects EF at G, then which of the following is correct?**

a. EF = 2AB
b. 9EF = 17AB
c. 9EF = 16AB
d. 9EF = 19AB

5. **ABCD is a parallelogram in the given figure. AB is divided at P and CD at Q such that AP : PB = 5 : 2 and CQ : QD = 3 : 1. If PQ meets AC at R, then which of the following is true?**



a. AR = $^{21}/_{40}$AC
b. AR = $^{21}/_{41}$AC
c. AR = $^{20}/_{41}$AC
d. AR = 23⁄40AC

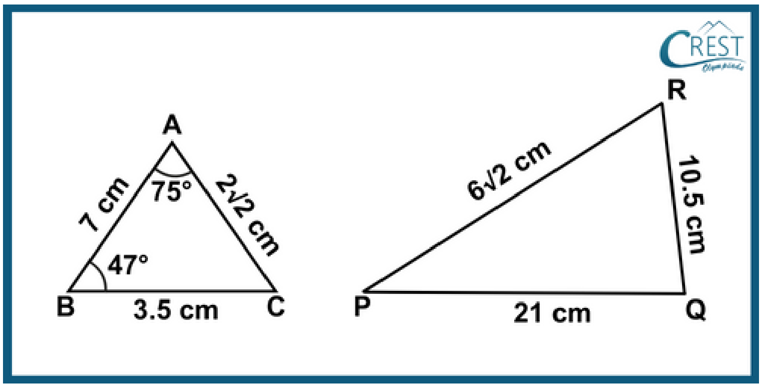
Answer Key

1. c - 58°

Explanation: We are given that in △ABC, AB = 7 cm, BC = 3.5 cm, CA = 2√2 cm, ∠A = 75° and ∠B = 47°.

In △PQR, PQ = 21 cm, QR = 10.5 cm, RP = 6√2 cm.

The figure is shown below:



From the similarity criterion, we have

$$\frac{AB}{DE} = \frac{7}{21} = \frac{1}{3}$$

$$\frac{BC}{QR} = \frac{3.5}{10.5} = \frac{1}{3}$$

$$\frac{CA}{RP} = \frac{2\sqrt{2}}{6\sqrt{2}} = \frac{1}{3}$$

$$\rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

According to SSS (Side-Side-Side) Similarity Criterion,

$\triangle ABC \sim \triangle PQR$

Thus, $\angle C = \angle P$ [Corresponding angles of the similar triangles]

We know that the sum of all angles of a triangle is 180°.

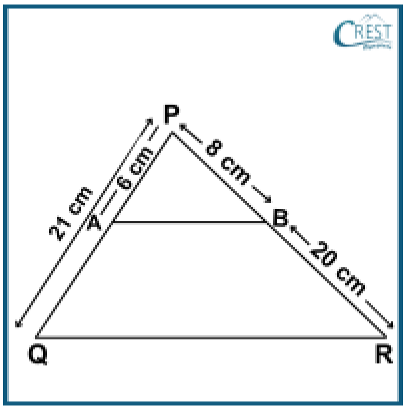
In $\triangle ABC$,
$\angle A + \angle B + \angle C = 180°$
$75° + 47° + \angle C = 180°$
$122° + \angle C = 180°$
$\angle C = 180° - 122°$
$\angle C = 58°$

Since $\angle C = \angle P$
$\angle P = 58°$

2. a – Only (i)

Explanation:

Consider Statement (i): We are given PQ = 21 cm, PA = 6 cm, BR = 20 cm and PB = 8 cm



Thus, AQ = PQ − PA
AQ = 21 − 6
AQ = 15 cm

Now PA/AQ = 6/15 = 2/5
Also, PB/BR = 8/20 = 2/5
PA/AQ = PB/BR

The converse of basic proportionality theorem states that if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
Thus, AB || QR

Thus, statement (i) is TRUE.

Consider Statement (ii):

We are given that in $\triangle ABC$ and $\triangle PQR$, $\angle A = 38°$, $\angle B = 57°$, $\angle P = 85°$ and $\angle Q = 57°$.

We know that the sum of all angles of a triangle is 180°.

Thus, in $\triangle ABC$,
$\angle A + \angle B + \angle C = 180°$
$38° + 57° + \angle C = 180°$
$95° + \angle C = 180°$
$\angle C = 180° - 95°$
$\angle C = 85°$

Now in $\triangle PQR$,
$\angle P + \angle Q + \angle R = 180°$
$85° + 57° + \angle R = 180°$
$142° + \angle R = 180°$
$\angle R = 180° - 142°$
$\angle R = 38°$

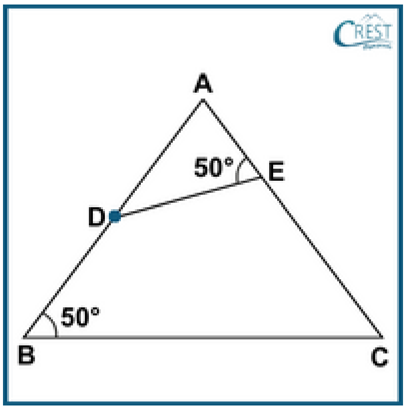
Thus, in $\triangle BAC$ and $\triangle QRP$,
$\angle A = \angle R = 38°$
$\angle B = \angle Q = 57°$
$\angle C = \angle P = 85°$
Thus, by AAA similarity
$\triangle BAC \sim \triangle QRP$

Thus, statement (ii) is FALSE.

Therefore, Only (i) is TRUE.

3. d - 11.8 cm

Explanation: The given figure is shown as:



Let ∠A = θ

In △ABC and △AED,
∠BAC = ∠DAE (Common)
∠ABC = ∠AED = 50° (Given)

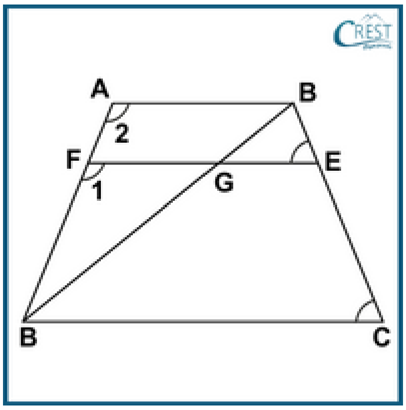
Thus, by AA similarity criterion,
△ABC ~ △AED
AC / AD = AB / AE = BC / ED

Taking AC / AD = AB / AE
AC / 7 = 12 / 5
AC = (12 × 7) / 5
AC = 84 / 5

Now, AC = AE + EC
EC = AC − AE
= (84 / 5) − 5
= (84 − 25) / 5
= 59 / 5
= 11.8 cm
∴ **EC = 11.8 cm**

4. b - 9EF = 17AB

Explanation: The figure is shown below:



We are given that DC = 3AB and BE/EC **= 4/5**

In $\triangle$DFG and $\triangle$DAB,

$\angle$DFG = $\angle$DAB [Corresponding angles since AB || FG]

$\angle$FDG = $\angle$ADB [Common]

We know that if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar by AA similarity criteria.

$\triangle DFG \sim \triangle DAB$ [By AA similarity]

Thus, $\frac{DF}{DA} = \frac{FG}{AB}$ … (1)

In trapezium ABCD, AB || EF || CD

$\frac{AF}{FD} = \frac{BE}{EC}$

But $\frac{BE}{EC}$ **=** $\frac{4}{5}$

$\frac{AF}{FD}$ **=** $\frac{4}{5}$

Adding 1 on both sides

$\frac{AF}{FD} + 1 = \frac{4}{5} + 1$

(AF + FD)/FD = (4 + 5)/5

$\frac{AD}{FD} = \frac{9}{5}$

$\frac{FD}{AD} = \frac{5}{9}$ … (2)

From equation (1) and (2),

$\frac{FG}{AB} = \frac{5}{9}$

FG = $\frac{5}{9}$AB … (3)

In $\triangle$BEG and $\triangle$BCD,

$\angle$BEG = $\angle$BCD [Corresponding angles since EG || CD]

$\angle$GBE = $\angle$DBC [Common]

We know that if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar by AA similarity criteria.

$\triangle BEG \sim \triangle BCD$ [By AA similarity]

Thus, $\frac{BE}{BC} = \frac{EG}{CD}$

We know that $\frac{BE}{EC}$ **=** $\frac{4}{5}$

$\frac{EC}{BE} = \frac{5}{4}$

Adding 1 on both sides
(EC/BE) + 1 = (5/4) + 1
(EC + BE)/BE = 5 + 44
$\frac{EC}{BE} = \frac{9}{4}$
$\frac{BE}{EC} = \frac{4}{9}$
But $\frac{BE}{BC} = \frac{EG}{CD}$
$\frac{EG}{CD} = \frac{4}{9}$
EG = $\frac{4}{9}$CD

We know that CD = 3AB
EG = $\frac{4}{9}$(3AB)
EG = $\frac{12}{9}$AB ... (4)

Adding equation (3) and (4),
FG + EG = $\frac{5}{9}$AB + $\frac{12}{9}$AB
EF = $\frac{17}{9}$AB
9EF = 17AB

5. c – AR = $\frac{20}{41}$AC

Explanation: In △APR and △CQR,

∠PAR = ∠QCR [Alternate interior angles since AB || CQ]
∠ARP = ∠CRQ [Vertically opposite angles]

We know that if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar by AA similarity criteria.
△APR ~ △CQR [By AA similarity]
$\frac{AP}{CQ} = \frac{PR}{QR} = \frac{AR}{CR}$ ... (1)

We are given that AP : PB = 5 : 2
AP/AB = AP/(AP + PB)
= 5/(5 + 2)
= 5/7
AP = $\frac{5}{7}$AB

We are given that CQ : QD = 3 : 1
CQ/CD = CQ/(CQ + QD)
= $\frac{3}{3+1}$
= $\frac{3}{4}$
CQ = $\frac{3}{4}$CD

Since ABCD is a parallelogram, its opposite sides are equal
AB = CD
CQ = $\frac{3}{4}$AB

From equation (1), we have
$\frac{AP}{CQ} = \frac{AR}{CR}$

Substituting AP = $\frac{5}{7}$AB and CQ = $\frac{3}{4}$AB in the above equation

$$\rightarrow \frac{\frac{5}{7}AB}{\frac{3}{4}AB} = \frac{AR}{CR}$$

$$\rightarrow \frac{5 \times 4}{7 \times 3} = \frac{AR}{CR}$$

$$\rightarrow \frac{AR}{CR} = \frac{20}{21}$$

$$\rightarrow \frac{CR}{AR} = \frac{21}{20}$$

Addaing 1 on both sides

$$\rightarrow \frac{CR}{AR} + 1 = \frac{21}{20} + 1$$

$$\rightarrow \frac{CR + AR}{AR} = \frac{21 + 20}{20}$$

$$\rightarrow \frac{AC}{AR} = \frac{41}{20}$$

$$\rightarrow \frac{AR}{AC} = \frac{20}{41}$$

$$\rightarrow AR = \frac{20}{41} AC$$

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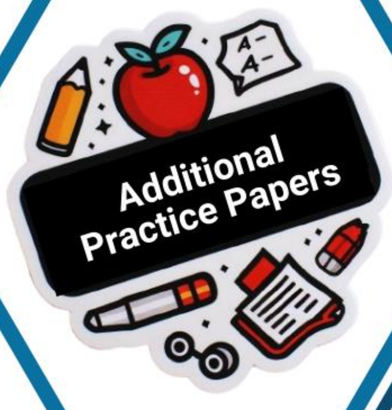

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