#CRESTInnovator

CREST Mathematics Olympiad (CMO) Worksheet for Class 10

Topic

Surface, Areas and Volumes

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Worksheet on Surface, Areas and Volumes

1. **The radius of the sphere, cone and cylinder is equal. Their total surface area is also equal. What is the ratio of their heights?**

   a. $1 : \sqrt{2} : 1$
   b. $2 : 2 : 1$
   c. $2 : \sqrt{2} : 1$
   d. $2 : 2\sqrt{2} : 1$

2. **A conical cup is filled with ice cream. The ice cream forms a hemispherical shape on its open top. The height of the hemispherical part is 4.2 cm. The height of the cone is equal to the radius of the hemispherical part. What is the volume of the ice cream?**

   a. 232.748 cm^3
   b. 232.648 cm^3
   c. 232.848 cm^3
   d. 232.548 cm^3

3. **If a solid metallic sphere of radius 9 cm is melted and recast into a right circular cone of base radius 4 cm, then what is the height of the cone?**

   a. 180.25 cm
   b. 182.25 cm
   c. 183.25 cm
   d. 181.25 cm

4. **If the area of the base of the frustum of a right circular cone is 25π cm^2, the diameter of the circular upper surface is 6 cm and the slant height is 8 cm, then what will be the total surface area of the frustum?**

   a. 308 cm^2
   b. 307 cm^2
   c. 304 cm^2
   d. 306 cm^2

5. **If a solid sphere of radius 7 cm is melted and moulded into small identical cubes of side length 2 cm, then how many such cubes can be formed from the sphere?**

   a. 178
   b. 181
   c. 180
   d. 179

Answer Key

1. d - $2 : 2\sqrt{2} : 1$

Explanation: Let the radius of the sphere, cone and cylinder be R.
Let the height of the sphere be H_1.
Let the height of the cone be H_2.
Let the height of the cylinder be H_3.
Let l be the slant height of the cone.

We know that the height of the sphere is the same as the diameter of the sphere.
$\mathbf{H_1 = 2R}$

We know that
Total Surface Area of the sphere = $4\pi R^2$
Total Surface Area of the cone = $\pi R(l + R)$
Total Surface Area of the cylinder = $2\pi R(H_3 + R)$

We are given that the total surface area of the sphere, cone and cylinder is equal.
So first taking,

Total surface area of the sphere = Total surface area of the cylinder
$4\pi R^2 = 2\pi R(H_3 + R)$
$2R = H_3 + R$
$\mathbf{R = H_3}$

Now taking,
Total surface area of the sphere = Total surface area of the cone
$4\pi R^2 = \pi R(l + R)$
$4R = l + R$
$\mathbf{3R = l \ldots (1)}$

In a cone,
$(\text{Slant height})^2 = \text{Height}^2 + \text{Radius}^2$
$(l)^2 = (H_2)^2 + (R)^2$
$(3R)^2 = (H_2)^2 + R^2$
$9R^2 = (H_2)^2 + R^2$
$9R^2 - R^2 = (H_2)^2$
$8R^2 = (H_2)^2$
$H_2 = \sqrt{8R^2}$
$\mathbf{H_2 = 2\sqrt{2}\,R}$

Now,
$H_1 : H_2 : H_3 = 2R : 2\sqrt{2}\,R : R$
$= 2 : 2\sqrt{2} : 1$

2. c - 232.848 cm^3

Explanation: We know that the volume of a hemisphere = $2\pi r^3 / 3$
Volume of a cone = $\pi r^2 h / 3$
We are given the height of the hemispherical part = 4.2 cm
Height of the hemispherical part = Radius of the hemisphere
The radius of the hemisphere = 4.2 cm

We are given that the height of the cone is equal to the radius of the hemispherical part.
Height of the cone = 4.2 cm

Now,
Volume of the ice cream = Volume of the hemispherical part + Volume of the cone
$= 2\pi r^3/3 + \pi r^2 h/3$
$= [\frac{2}{3} \times \frac{22}{7} (4.2)^3] + [\frac{1}{3} \times \frac{22}{7} \times (4.2)^2 \times 4.2]$
$= [\frac{2}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2] + [\frac{1}{3} \times \frac{22}{7} \times (4.2)^2 \times 4.2]$
$= (2 \times 22 \times 1.4 \times 0.6 \times 4.2) + (22 \times 1.4 \times 0.6 \times 4.2)$
$= (155.232) + (77.616)$
$= 232.848\ cm^3$

3. b - 182.25 cm

Explanation: We know that the volume of a sphere = $4\pi r^3/3$
Volume of a cone = $\pi r^2 h / 3$
The radius of the metallic sphere = 9 cm
The radius of the cone = 4 cm

Now, Volume of the metallic sphere = $4\pi (9)^3 / 3$
$= 4\pi (729) / 3$
$= 4\pi (243)$
$= 972\pi$
Volume of the cone = $\pi(4)^2 h / 3$
$= 16\pi h / 3$

When one solid is melted and recast into another shape, its volume is the same.
→ Volume of the cone = Volume of the metallic sphere
→ $16\pi h / 3 = 972\pi$
→ $h = \frac{972\pi \times 3}{16\pi}$
→ $h = \frac{243 \times 3}{4}$
→ $h = 729 / 4$
→ $h = 182.25$ cm

4. a - 308 cm^2

Explanation: Let the radius of the base of the frustum be R cm.
We are given that the area of the base of the frustum is $25\pi\ cm^2$.

We know that the area of the base is πr^2

$\pi R^2 = 25\pi$
$R^2 = 25$
$R = \sqrt{25}$
$R = 5$ cm

We are given the diameter of the circular upper surface = 6 cm
The radius of the circular upper surface (r) = 3 cm
We are given that the slant height (l) = 8 cm

We know that the total surface area of the frustum = $\pi R^2 + \pi r^2 + \pi l(R + r)$
→ Total surface area of frustum = $\frac{22}{7} \times (5)^2 + \frac{22}{7} \times (3)^2 + \frac{22}{7} \times 8 \times (5 + 3)$
$= \frac{22}{7} \times 25 + \frac{22}{7} \times 9 + \frac{22}{7} \times 8 \times 8$
$= \frac{22}{7} \times (25 + 9 + 64)$
$= \frac{22}{7} \times 98$
$= 22 \times 14$
$= 308$ cm^2

5. d – 179

Explanation: We know that the volume of a sphere = $4\pi r^3 / 3$

Volume of the cube = $(\text{side})^3$

Volume of the sphere = $4\pi r^3 / 3$
$= 4\pi(7)^3 / 3$
$= (4 \times (22 / 7) \times (7)^3 / 3$
$= (4 \times 22 \times (7)^2 / 3$
$= 4312 / 3$
$= 1437.33$ cm^3

Volume of the cube = $(\text{side})^3$
$= (2)^3$
$= 8$ cm^3

When one solid is melted and recast into another shape, its volume is the same.
Thus, Number of cubes formed = (Volume of the sphere) / (Volume of the cube)
$= 1437.33 / 8$
$= 179.67$
~ 179

Thus, 179 identical cubes of side 2 cm can be formed from the solid sphere with a radius of 7 cm when melted and moulded.

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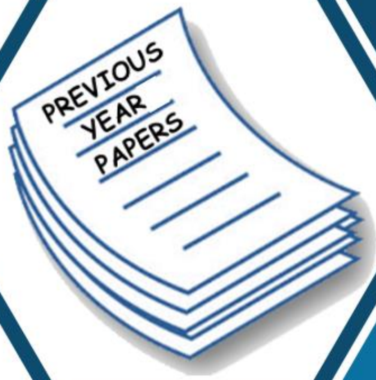

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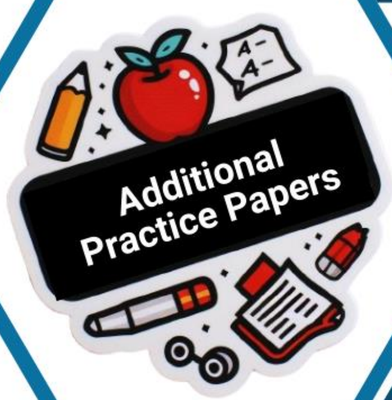

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