#CRESTInnovator

CREST Mathematics Olympiad (CMO) Worksheet for

Class 10

Topic

Quadratic and Linear Equations

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Worksheet on Quadratic and Linear Equations

1. What is the solution of the following pair of linear equations $5x + y = 10$ and $10x - 7y = 50$?

a. $x = -\frac{8}{3}$ and $y = \frac{10}{3}$
b. $x = -\frac{8}{3}$ and $y = -\frac{10}{3}$
c. $x = \frac{8}{3}$ and $y = \frac{10}{3}$
d. $x = \frac{8}{3}$ and $y = -\frac{10}{3}$

2. If and are the roots of the equation $x^2 + 6x + 8 = 0$, then which of the following equations has roots $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$?

a. $x^2 - 144x + 40 = 0$
b. $x^2 + 144x - 40 = 0$
c. $x^2 + 40x - 144 = 0$
d. $x^2 - 40x + 144 = 0$

3. The ratio between a two-digit number and the sum of digits of that number is 5 : 2. If the digit in the unit place is 4 more than the digit in the tenth place, then what is the number?

a. 25
b. 18
c. 15
d. 38

4. For which values of p and q, will the following pair of linear equations

$3x + 2y = 7$ and $(3p + 10q)x + (p + 4q)y = 2p - q + 1$ have infinitely many solutions?

a. p = - 8/33 and q = 1/11
b. p = - 8/11 and q = 1/11
c. p = 8/33 and q = - 1/11
d. p = 8/11 and q = - 1/11

5. What is the value of 'p' if the product of roots of the given quadratic equation $(p + 7)x^2 - (2p - 11)x + (3p - 4) = 0$ is $\frac{13}{5}$?

a. 45.5
b. 55.5
c. 65.5
d. 75.5

Answer Key

1. d - $x = 8/3$ and $y = -\frac{10}{3}$

Explanation: We are given a pair of linear equations in two variables

$5x + y = 10$ … (1)
$10x - 7y = 50$ … (2)

Now, express the value of y in terms of x from the equation (1),
$\mathbf{y = 10 - 5x}$ … (3)

Now we substitute this value of **y** in Equation (2),
$\rightarrow 10x - 7(10 - 5x) = 50$
$\rightarrow 10x - 70 + 35x = 50$
$\rightarrow 45x = 50 + 70$
$\rightarrow 45x = 120$
$\rightarrow x = 120/45$
$\rightarrow x = 8/3$

Substitute this value of $\mathbf{x = 8/3}$ in equation (3) to find the value of y.
$\rightarrow y = 10 - 5(\frac{8}{3})$
$\rightarrow y = 10 - \frac{40}{3}$
$\rightarrow y = \frac{30 - 40}{3}$
$\rightarrow y = -\frac{10}{3}$

Thus, $x = 8/3$ and $y = -10/3$

2. d - $x^2 - 40x + 144 = 0$

Explanation: We are given $x^2 + 6x + 8 = 0$

Here, $a = 1$, $b = 6$ and $c = 8$

We know that the roots of the quadratic equation are given by

$$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\rightarrow x = \frac{-6 + \sqrt{[(6)^2 - 4(1)(8)]}}{2(1)} \text{ and } x = \frac{-6 - \sqrt{[(6)^2 - 4(1)(8)]}}{2(1)}$$

$$\rightarrow x = \frac{-6 + \sqrt{(36) - (32)}}{2} \text{ and } x = \frac{-6 - \sqrt{(36) - (32)}}{2}$$

$$\rightarrow x = \frac{-6 + \sqrt{4}}{2} \text{ and } x = \frac{-6 - \sqrt{4}}{2}$$

$$\rightarrow x = \frac{-6 + 2}{2} \text{ and } x = \frac{-6 - 2}{2}$$

$$\rightarrow x = \frac{-4}{2} \text{ and } x = \frac{-8}{2}$$

$$\rightarrow x = -2 \text{ and } x = -4$$

Thus, $\alpha = -2$ and $\beta = -4$ are the roots of the given quadratic equation.

Now, $\alpha + \beta = -2 + (-4) = -2 - 4 = -6$
$\rightarrow (\alpha + \beta)^2 = (-6)^2 = 36$
Also, $\alpha - \beta = -2 - (-4) = -2 + 4 = 2$
$\rightarrow (\alpha - \beta)^2 = (2)^2 = 4$

We know that a quadratic equation is obtained by the formula
$x^2 -$ (Sum of roots)x + Product of roots = 0

Thus, the quadratic equation with $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ as roots is:
$\rightarrow x^2 - [(\alpha + \beta)^2 + (\alpha - \beta)^2]x + [(\alpha + \beta)^2 \times (\alpha - \beta)^2] = 0$
$\rightarrow x^2 - [36 + 4]x + [36 \times 4] = 0$
$\rightarrow x^2 - 40x + 144 = 0$

3. c - 15

Explanation: Let the digit in the tenth place be a and the digit in the unit place be b.

Hence, the number is 10a + b.

We are given that the ratio between a two-digit number and the sum of digits of that number is 5 : 2.
$\rightarrow \frac{10a + b}{a+b} = \frac{5}{2}$
$\rightarrow 2(10a + b) = 5(a + b)$
$\rightarrow 20a + 2b = 5a + 5b$
$\rightarrow 20a - 5a = 5b - 2b$
$\rightarrow 15a = 3b$
$\rightarrow b = 15a3$
$\rightarrow b = 5a$ … (1)

We are also given that the digit in the unit place is 4 more than the digit in the tenth place.
→ b = a + 4 … (2)

Now, substitute the value of b = 5a in equation (2),
→ 5a = a + 4
→ 5a − a = 4
→ 4a = 4
→ a = 44
→ a = 1

Putting the value of a in equation (1) to find the value of b,
→ b = 5(1)
→ b = 5

Thus, the required number = 10a + b = 10(1) + 5 = 15

4. a - p = - 8/33 and q = 1/11

Explanation: We are given the pair of linear equations

3x + 2y = 7

(3p +10q)x + (p + 4q)y = 2p − q + 1

Here, $a_1 = 3$, $a_2 = 3p + 10q$, $b_1 = 2$, $b_2 = p + 4q$, $c_1 = 7$ and $c_2 = 2p - q + 1$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Thus, $\frac{3}{3p + 10q} = \frac{2}{p + 4q} = \frac{7}{2p - q + 1}$

$\Rightarrow \frac{3}{3p + 10q} = \frac{2}{p + 4q}$ **and** $\frac{3}{3p + 10q} = \frac{7}{2p - q + 1}$

First solving,

$$\left(\frac{3}{3p + 10q}\right) = \left(\frac{2}{p + 4q}\right)$$

→ 3(p + 4q) = 2(3p + 10q)
→ 3p + 12q = 6p + 20q
→ 3p − 6p = 20q − 12q
→ − 3p = 8q
→ $p = \frac{-8}{3}q$ … (1)

Now solving,

$$\left(\frac{3}{3p + 10q}\right) = \left(\frac{7}{2p - q + 1}\right)$$

$\rightarrow$ 3 (2p − q + 1) = 7(3p + 10q)
$\rightarrow$ 6p − 3q + 3 = 21p + 70q
$\rightarrow$ 6p − 21p + 3 = 70q + 3q
$\rightarrow$ − 15p + 3 = 73q … (2)

Substituting the value of p in equation (2)
$\rightarrow - 15(\frac{-8}{3} q) + 3 = 73q$
$\rightarrow - 5(-8q) + 3 = 73q$
$\rightarrow 40q + 3 = 73q$
$\rightarrow 40q - 73q = -3$
$\rightarrow - 33q = -3$
$\rightarrow q = \frac{-3}{-33}$
$\rightarrow q = \frac{1}{11}$

Now put the value of q in equation (1) to find the value of p
$\rightarrow p = \frac{-8}{3} q$
$\rightarrow p = \frac{-8}{3} (\frac{1}{11})$
$\rightarrow p = \frac{-8}{33}$

Hence, $p = \frac{-8}{33}$ and $q = \frac{1}{11}$.

5. b – 55.5

Explanation: We are given the quadratic equation $(p + 7)x^2 - (2p - 11)x + (3p - 4) = 0$

Here a = p + 7, b = − (2p − 11) and c = 3p − 4

We know that the product of roots of a quadratic equation = $\frac{c}{a}$

Product of the roots of the given quadratic equation = $\frac{13}{5}$
$\rightarrow \left(\frac{3p - 4}{p + 7}\right) = \frac{13}{5}$
$\rightarrow 13 \times (p + 7) = 5 \times (3p - 4)$
$\rightarrow 13p + 91 = 15p - 20$
$\rightarrow 91 + 20 = 15p - 13p$
$\rightarrow 111 = 2p$
$\rightarrow p = \frac{111}{2}$
$\rightarrow p = 55.5$

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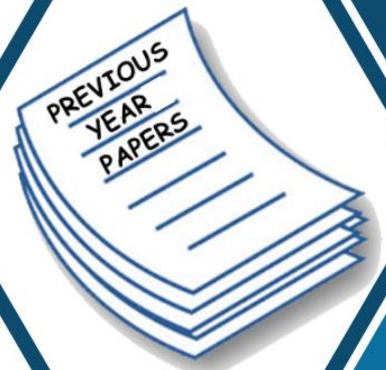

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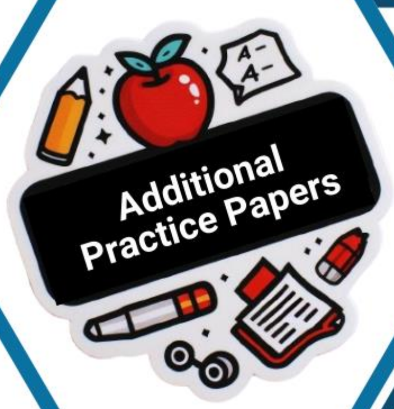

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