#CRESTInnovator

CREST Mathematics Olympiad (CMO) Worksheet for Class 10

Topic

Coordinate Geometry

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Worksheet on Coordinate Geometry

1. **Which type of triangle is formed using the vertices A (1, 2), B (−3, 12) and C (−5, −6)?**

   a. Isosceles triangle
   b. Equilateral triangle
   c. Scalene triangle
   d. Right-angled triangle

2. **The point which divides the line segment joining the points (5, –9) and (–2, 5) in ratio 2 : 3 internally lies in which quadrant?**

   a. First quadrant
   b. Second quadrant
   c. Third quadrant
   d. Fourth quadrant

3. **If the point P (5, 4) lies on the line segment joining points A (3, 2) and B (9, 8), then which of the following is true?**

   a. BP = AB/3
   b. AP = BP/2
   c. AP = BP/3
   d. BP = AB/2

4. **Out of the following points, which three points satisfy the condition of collinearity?**

   a. (5, −3), (4, 1) and (−2, 7)
   b. (1, 2), (3, −2) and (0, 5)
   c. (−4, −1), (2, 3) and (−3, 6)
   d. (−6, 10), (−4, 6) and (3, −8)

5. **In what ratio does the point (–2, 3) divide the line segment joining the points A (–8, 6) and B (5, –9)?**

   a. 7 : 5
   b. 7 : 6
   c. 5 : 7
   d. 6 : 7

Answer Key

1. c - Scalene triangle

   Explanation: We know that:

   → An equilateral triangle has all sides equal.
   → An isosceles triangle has two sides equal.

→ A scalene triangle has no sides equal.
→ A right-angled triangle satisfies the Pythagoras Theorem.

We can find the lengths of the sides of a triangle using the distance formula.

Distance between any two points (x_1, y_1) and (x_2, y_2) is given by:
$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

We are given △ABC with vertices A (1, 2), B (−3, 12) and C (−5, −6).

Now, using the distance formula, find the length of AB, BC and CA.
→ $AB = \sqrt{[((-3) - 1)^2 + (12 - 2)^2]}$
$= \sqrt{[(-4)^2 + (10)^2]}$
$= \sqrt{[16 + 100]}$
$= \sqrt{116}$
$= 2\sqrt{29}$ units

→ $BC = \sqrt{[((-5) - (-3))^2 + ((-6) - 12)^2]}$
$= \sqrt{[(-5 + 3)^2 + (-6 - 12)^2]}$
$= \sqrt{[(-2)^2 + (-18)^2]}$
$= \sqrt{[4 + 324]}$
$= \sqrt{328}$
$= 2\sqrt{82}$ units

→ $CA = \sqrt{[((-5) - 1)^2 + ((-6) - 2)^2]}$
$= \sqrt{[(-6)^2 + (-8)^2]}$
$= \sqrt{[36 + 64]}$
$= \sqrt{100}$
$= 10$ units

Now, $AB^2 = (\sqrt{116})^2$
$= 116$

$BC^2 = (\sqrt{328})^2$
$= 328$

$CA^2 = (10)^2$
$= 100$

Thus, $AB^2 + CA^2 \neq BC^2$

Thus, it does not satisfy the Pythagoras theorem.

Since the length of all three sides is different and it doesn't satisfy the Pythagoras theorem, thus △ABC is a scalene triangle.

2. d - Fourth quadrant

Explanation: Here,

$x_1 = 5$
$x_2 = -2$
$y_1 = -9$
$y_2 = 5$
$m = 2$
$n = 3$

Let P (x, y) divide the line segment joining the points (5, – 2) and (− 9, 5) in the ratio 2 : 3 internally. Thus the coordinates of P are

$$\rightarrow P(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$P(x,y) = \left(\frac{2(-2)+3}{(5)2+3}, \frac{2(5)+3(-9)}{2+3}\right)$$

$$= \left(\frac{-4+15}{5}, \frac{10-27}{5}\right)$$

$$= \left(\frac{11}{5}, \frac{-17}{5}\right)$$

Thus $P(x, y) = \left(\frac{11}{5}, \frac{-17}{5}\right)$ divides the line segment internally.

We know that x is positive and y is negative in the fourth quadrant.

Thus, $P\left(\frac{11}{5}, \frac{-17}{5}\right)$ lies in the fourth quadrant.

3. b - AP = BP/2

Explanation: We are given that point P (5, 4) lies on the line segment joining points A (3, 2) and B (9, 8).

Thus,

$AP = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
$= \sqrt{[(5 - 3)^2 + (4 - 2)^2]}$
$= \sqrt{[(2)^2 + (2)^2]}$
$= \sqrt{[4 + 4]}$
$= \sqrt{8}$
$= 2\sqrt{2}$

$BP = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
$= \sqrt{[(5 - 9)^2 + (4 - 8)^2]}$

$= \sqrt{[(-4)^2 + (-4)^2]}$
$= \sqrt{[16 + 16]}$
$= \sqrt{32}$
$= 4\sqrt{2}$

$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
$= \sqrt{[(9 - 3)^2 + (8 - 2)^2]}$
$= \sqrt{[(6)^2 + (6)^2]}$
$= \sqrt{[36 + 36]}$
$= \sqrt{72}$
$= 6\sqrt{2}$

We know that: $AP = 2\sqrt{2}$ and $BP = 4\sqrt{2}$

$\therefore AP = BP/2$

4. d - (−6,10), (−4, 6) and (3, −8)

Explanation: We know that the area of a triangle formed by three collinear points is zero.

Also, If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of an △ABC, then its area is given by:

Area $= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
Option a)

We are given three points (5, −3), (4, 1) and (−2, 7)
Here,
$x_1 = 5$
$x_2 = 4$
$x_3 = -2$
$y_1 = -3$
$y_2 = 1$
$y_3 = 7$

Thus, Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
$= \frac{1}{2} [5(1 - 7) + 4(7 - (-3)) + (-2)((-3) - 1)]$
$= \frac{1}{2} [(5)(-6) + 4(10) + (-2)(-4)]$
$= \frac{1}{2} [-30 + 40 + 8]$
$= \frac{1}{2} [18]$
$= 9$
$\neq 0$

Thus, the points are not collinear.

Option b)
We are given three points (1, 2), (3, −2) and (0, 5)

Here,

$x_1 = 1$
$x_2 = 3$
$x_3 = 0$
$y_1 = 2$
$y_2 = -2$
$y_3 = 5$

Thus, Area of triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_1(y_3 - y_1) + x_3(y_1 - y_2)]$
$= \frac{1}{2}[1(-2 - 5) + 3(5 - 2) + 0(2 - (-2))]$
$= \frac{1}{2}[1(-7) + 3(3) + 0(2 + 2)]$
$= \frac{1}{2}[-7 + 9 + 0]$
$= \frac{1}{2}[2]$
$= 1$
$\neq 0$

Thus, the points are not collinear.

Option c)

We are given three points (−4, −1), (2, 3) and (−3, 6)

Here,
$x_1 = -4$
$x_2 = 2$
$x_3 = -3$
$y_1 = -1$
$y_2 = 3$
$y_3 = 6$

Thus, Area of triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
$= \frac{1}{2}[(-4)(3 - 6) + 2(6 - (-1)) + (-3)((-1) - 3)]$
$= \frac{1}{2}[(-4)(-3) + 2(7) + (-3)(-4)]$
$= \frac{1}{2}[12 + 14 + 12]$
$= \frac{1}{2}[38]$
$= 19$
$\neq 0$

Thus, the points are not collinear.

Option d)

We are given three points (− 6, 10), (−4, 6) and (3, −8)

Here,
$x_1 = -6$
$x_2 = -4$
$x_3 = 3$
$y_1 = 10$
$y_2 = 6$
$y_3 = -8$

Thus, Area of triangle = ½[$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]
= ½[(−6)(6 − (−8)) + (−4)(−8 − 10) + 3(10 − 6)]
= ½[(−6)(14) + (−4)(−18) + 3(4)]
= ½[−84 + 72 + 12]
= ½[−84 + 84]
= ½[0]
= 0

So, the area of the triangle is zero. Thus, the points are collinear.

5. d – 6 : 7

Explanation: Let point (–2, 3) divide the line segment joining the points A(–8, 6) and B(5, –9) in k : 1 ratio.

We know that if the point M (x, y) divides the line segment joining P (x_1, y_1) and Q (x_2, y_2) internally in the ratio m : n, then the coordinates of M are given by the section formula as

$$M(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

(–2, 3) = (**5k - 8/k + 1, - 9k + 6/k + 1**)

We know that if (x, y) = (a, b) then x = a and y = b

–2 = **(5k - 8/k + 1)** and 3 = **(-9k + 6/k + 1)**

Using –2 = 5k - 8k + 1 to find the ratio.

⇒ –2 = **(5k - 8/k + 1)**
⇒ –2 (k + 1) = 5k – 8
⇒ –2k – 2 = 5k – 8
⇒ –2k – 5k = – 8 + 2
⇒ –7k = – 6
⇒ 7k = 6
⇒ k = 6/7

Ratio = k : 1 = 6/7 : 1 = 6 : 7

Thus, point (– 2, 3) divides the line segment joining the points A(– 8, 6) and B(5, – 9) in the ratio 6 : 7.

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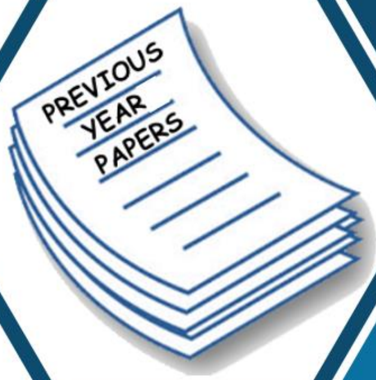

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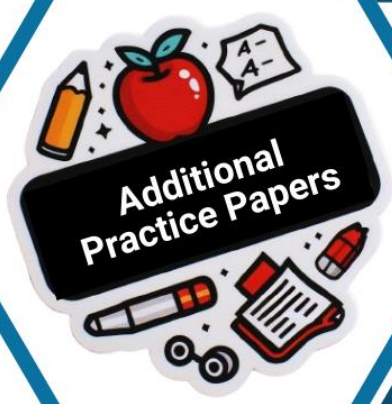

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