



**Stage IV**



Grades 9-10

CREST Teacher Mathematics Olympiad (CTMO)

# Sample Paper

## Pattern and Marking Scheme

Stage	Topic/Section	No. of Questions	Marks per Question	Total Marks
For stage IV (Grades 9-10)	Practical Mathematics	30	3	90
	Achiever's Section	20	6	120
<b>Grand Total</b>		<b>50</b>		<b>210</b>

The total duration of the exam is 60 minutes.

**Note:** For every incorrect answer, there's a penalty of 1/3rd of the total marks allotted to that question.

## Syllabus

Number system, Expressions and equations, Comparing quantities, Coordinate geometry, Trigonometry, Geometry, Statistics and data handling

For more details visit: <https://www.crestolympiads.com/teacher-mathematics-olympiad>

## Practical Mathematics (Each Question is 3 Marks)

- A group of students in a class collects money. The amount contributed by each student is equivalent to the number of students in the class. One-third of the total collected amount is spent in the farewell party and the left amount is \$5766. Total how many students are there in the class?
  - 93
  - 97
  - 99
  - 102
- Sia thought about the positive integer  $n$ , divided it by 9, and found the remainder to be 7. What will be the remainder if divided  $(3n - 1)$  again by 9?
  - 2
  - 3
  - 4
  - 5
- What is the ratio of the sum and product of the roots of the equation  $7x^2 - 12x + 18 = 0$ ?
  - 6 : 11
  - 2 : 3
  - 1 : 11
  - 12 : 15
- Rex asked Tom to find the original number. Tom discovered that a rational number that if he subtracts 3 from the numerator and adds 2 to the denominator, the new number that results is  $\frac{1}{5}$ . What is the original number?
  - $\frac{5}{8}$
  - $1\frac{3}{5}$
  - $\frac{7}{8}$
  - $1\frac{1}{5}$
- Robert and his grandfather are having a conversation. By the end of 2017, Robert informed his grandfather, he would be half his grandfather's age. The total number of years they were born is 3854. How old will Robert be at the end of 2022?
  - 60 years
  - 62 years
  - 65 years
  - 66 years
- The King of the Mountains went up the valley at 15 km an hour and down it at 45 km an hour. It took him two hours in total. Assuming that the distance he travelled up and down are the same, how far is it from the bottom to the top of the valley?
  - 11.2 km
  - 22.5 km
  - 13 km
  - 13.2 km
- Sandra has a monthly habit of measuring and recording her height and weight. When she looked into it, she discovered that her height is increasing at a 2% rate when compared to the previous year. What was her height two years ago if she is now 156.06 cm tall?
  - 148 cm
  - 150 cm
  - 152 cm
  - 154 cm
- Max is a real estate agent. He bought a building for \$80,000, and due to the sudden drop in real estate values, its value depreciates by 20% every year, compared to its value at the start of the year. What is the profit made by selling the building he bought at the end of the second year for \$53,600?
  - \$2,400
  - \$2,300
  - \$2,200
  - \$2,100
- Aliz, Betty, and Cary competed in arm wrestling. Two girls wrestled in each game, while the third rested. After each game, the winner faced the girl who had rested in the following game. Aliz appeared 10 times, Betty appeared 15 times, and Cary appeared 17 times.

Who was the loser in the second game?

- a. Aliz
- b. Cary
- c. Betty
- d. No one loses the game

10. Amy, Diana, Tia, and Tom were in the Points (6, 8), (3, 7), (-2, -2) and (1, -1) respectively. when the line is drawn in such a way that a quadrilateral is formed What will the quadrilateral structure look like?

- a. Rhombus
- b. Square
- c. Rectangle
- d. Parallelogram

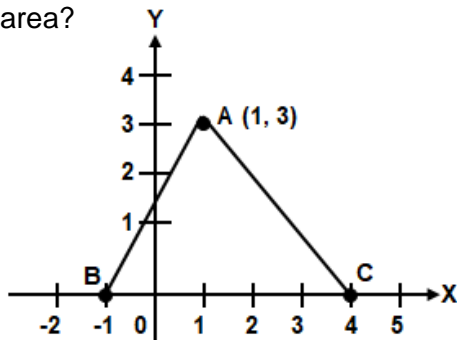
11. Find the coordinates of the point which divides the line segment joining the points (6, 3) and (-4, 5) in the ratio 3 : 2 internally:

- a. (24, 9)
- b. (-24, 9)
- c. (-24, -9)
- d. (24, -9)

12. For various vertices, Duke is attempting to calculate the area of a triangle. Max, a friend of his, has provided the vertices for it this time as (a, b + c), (a, b - c), and (-a, c). What is the area of the triangle?

- a. 2ab
- b. 2ac
- c. 2ab + c
- d. 2ac + b

13. On a graph paper, Frank has drawn the illustration below. What is the triangle's area?



- a. 15 sq. units
- b. 10 sq. units
- c. 7.5 sq. units
- d. 2.5 sq. units

14. Find the value of the following question.

$$\left(\frac{\sqrt{3}+2\cos A}{1-2\sin A}\right)^{-3} + \left(\frac{1+2\sin A}{\sqrt{3}-2\cos A}\right)^{-3} = \underline{\hspace{2cm}}$$

- a. 4
- b. 3
- c. 2
- d. 0

15. In a  $\triangle ABC$ , it is given that  $\angle C = 90^\circ$  and  $\tan A = \frac{1}{\sqrt{3}}$  then find the value of  $(\sin A \cos B + \cos A \sin B)$ .

- a. 1
- b. 1/2
- c. 3
- d. 0

16. If  $\sin x + \sin^2 x = 1$ , then  $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 2$  is equals to \_\_\_\_\_.

- a. 0
- b. 1
- c. -1
- d. 2

17. Lucia want to find out the value of A, B and C respectively when they are acute and If  $\sin(A + B + C) = 1$ , then  $\tan(A - B) = \frac{1}{\sqrt{3}}$  and  $\sec(A + C) = 2$ .

- a.  $0^\circ, 30^\circ, 60^\circ$
- b.  $60^\circ, 30^\circ, 0^\circ$
- c.  $30^\circ, 0^\circ, 60^\circ$
- d.  $0^\circ, 60^\circ, 30^\circ$

18. Given below are the steps of construction of a pair of tangents to a circle of radius 6 cm from a point on the concentric circle of radius 8 cm. Find which of the following steps is INCORRECT.

Steps of Construction:

Step I: Take a point O on the plane paper and draw a circle of radius  $OA = 6$  cm. Also, draw a concentric circle of radius  $OB = 8$  cm.

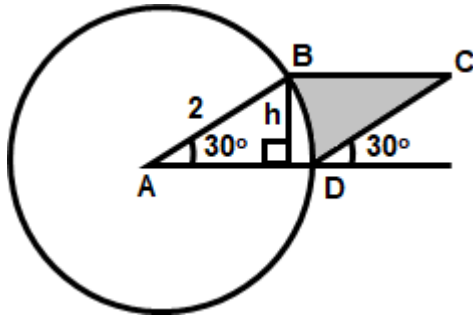
Step II: Find the mid-point A of OB and draw a circle of radius  $BA = AO$ . Suppose this circle intersects the circle of radius 6 cm at P and Q.

Step III: Join BP and BQ to get the desired tangents.

- a. Step I                      b. Step I and II  
c. Step II                      d. Step II and III

- a. 60.6                      b. 62.5  
c. 66.6                      d. 69.2

19. Shelin drew a circle with a radius of two units and a centre at A. Find the area of the shaded area if ABCD is a parallelogram.

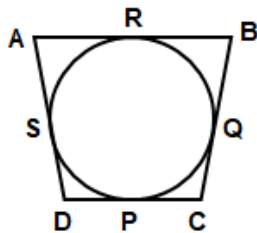


- a. 0.9528                      b. 0.9321  
c. 0.9177                      d. 0.9092

20. Duke determined that the perimeter of a sector of a circle with a radius of 5.2 cm is 16.4 cm. What would be the sector's area?

- a. 15.6 cm<sup>2</sup>                      b. 15.3 cm<sup>2</sup>  
c. 15.2 cm<sup>2</sup>                      d. 15.1 cm<sup>2</sup>

21. Leo drew a quadrilateral and drew a circle in it, as shown in the figure, where the circle touches all four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm, and CD = 4 cm. What is the value of AD?



- a. 10 cm                      b. 8 cm  
c. 3 cm                      d. 2 cm

22. Ashley is an expert in statistics. She was teaching her friend Reh about it. Ashley asked Reh what the upper-class boundary of the highest class is if the width of each of the twelve classes in a frequency distribution is 4.5 and the lower-class boundary of the lowest class is 12.6.

23. Stephen has five numbers. He stated that the mean of five numbers is 27. If I take out one number, the mean becomes 25. Determine the excluded number.

- a. 30                      b. 32  
c. 34                      d. 35

24. Mrs. Patrick teaches students about the concept of mean. She asked students to weigh themselves and write them down to help them understand. When they calculated the mean, they discovered the average weight of a class of 34 students is 46.5 kg. When Mrs. Patrick's weight is counted in, the mean rises by 500 g. Mrs. Patrick's weight?

- a. 64 kg                      b. 60 kg  
c. 52 kg                      d. 50 kg

25. Federick and Tom are playing snake and ladders. They both want to start first, so they planned on tossing a one-dollar coin three times and recording the results each time. Federick wins if all of the coin tosses come up with the same result, i.e. three heads or three tails; otherwise, Tom wins. Determine the probability that Tom will win the game.

- a.  $\frac{1}{4}$                       b.  $\frac{1}{2}$   
c.  $\frac{3}{4}$                       d.  $\frac{4}{5}$

26. The school has organised a quiz team competition, and there are five teams ready to go. Each team is made up of either all boys or all girls. The team members are 9, 15, 17, 19, and 21. After all members of the first team have begun, the number of girls who have not yet begun is three times that of boys who have not yet begun. How

many people are on the team that has already begun?

- a. 21
- b. 19
- c. 17
- d. 15

27. Gilbert and his team participated in an inter-school level cricket tournament. Because of their efforts, they made it to the finals. Gilbert batted first in the championship game. He played so well that he hit eight sixers out of 32 balls. Determine the probability that Gilbert has not hit a sixer in a ball.

- a. 0.25
- b. 0.75
- c. 0.92
- d. 0.98

28. What percentage of the total votes was received by the candidate who won the election, given that the three

candidates received 1136, 7636 and 11628 votes, respectively?

- a. 57%
- b. 60%
- c. 61%
- d. 63%

29. A lent Rs. 2500 to B and a certain amount to C simultaneously at a 7% annual simple interest rate. After 4 years, A received a total interest of \$1120 from both B and C. Find the amount lent to C.

- a. \$1500
- b. \$1800
- c. \$2000
- d. \$2400

30. If one of the solutions of the quadratic equation  $x^2 - kx - 15 = 0$  is 2, what is the value of 'k'?

- a.  $-11/2$
- b.  $2/3$
- c.  $1/8$
- d.  $-3/5$

## Achievers' Section (Each Question is 6 Marks)

31. To gain 25% after allowing a discount of 10%, what is the shopkeeper must mark the price of the article which costs him \$36?

- a. \$35
- b. \$40
- c. \$45
- d. \$50

32. The fourth vertex of the square whose consecutive vertices are (2, 1), (7, 4), and (-2, 5) is

- a. (2,-2)
- b. (2,-1)
- c. (1,-2)
- d. (1,-1)

33. The mathematics, music, and art seminar applications are now being accepted at Cylvia International School. They received a sizable number of applications from various universities. Based on the submitted synopsis, they narrowed down the pool of applicants. There are 196, 168, and 392 participants in each of the three subjects that they called. How many

rooms are needed a minimum if the same number of participants are to be seated in each room and they are all considering the same topic?

- a. 22
- b. 25
- c. 27
- d. 29

34. Nike carries four integer flash cards labelled a, b, c, and d, which satisfy  $ab = 2cd$ . Which of the following numbers cannot be the value of the product  $abcd$ ?

- a. 50
- b. 100
- c. 200
- d. 4000 m

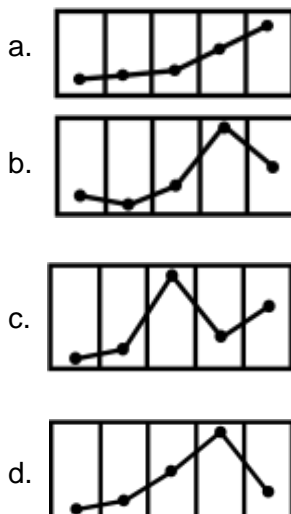
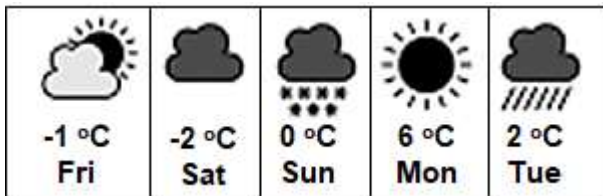
35. Two persons Austin and Harry joined D.W. Associates. Austin and Harry started with an initial salary of \$50,000 and \$64,000 respectively with annual increment of \$2,500 and \$2,000 each respectively. In which year will Austin start earning more salary than Harry?

- a. 28th year      b. 29th year  
c. 31st year      d. 32th year

**36.** Albert set out on a 520-kilometer drive with 14 litres of gas in the tank. His car uses one litre of fuel every ten kilometres. He reads a road sign after driving 55 kilometres that shows the distances from that point to five petrol stations ahead on the road. These are the distances: 35 km, 45 km, 55 km, 75 km, and 95 km. The car's fuel tank holds 40 litres, and Albert only wants to stop once to fill it. How far away is the gas station where he should stop?

- a. 34 km      b. 45 km  
c. 75 km      d. 100 km

**37.** Kathy travels for work and uses a weather app to learn about the forecasted weather. When she looks at her weather app, it displays the predicted weather and maximum temperatures for the next five days. Which of the following is the corresponding maximum temperature graph?



**38.** If the points (a, 0), (0, b), and (1, 1) are collinear, then  $(1/a) + (1/b) = 1$ .

- a. 2      b. 1  
c. -1      d. -2

**39.** While on vacation, Andrea flew in and landed on a tiny island in the middle of a 50-meter-wide river. There, she noticed a tall tree standing. where X and Y are points on the two banks that are parallel to the tree and directly across from one another. Find the height of the tree if the angles of elevation of the tree's top from X and Y are  $60^\circ$  and  $30^\circ$ , respectively.

- a. 21.65 m      b. 23.24 m  
c. 25.77 m      d. 27.21 m

**40.** A pole stands vertically on a riverbank. The angle of elevation of the top of the pole from a point on the opposite bank directly opposite the pole is  $60^\circ$ . The angle of elevation of the top of the pole is  $30^\circ$  from a point 20 metres away on the same bank. Determine the river's width.

- a. 8 cm      b. 10 cm  
c. 12 cm      d. 14 cm

**41.** The parrot and its family live in a tree, and she wants to protect its young ones because a snake is nearby. It is perched on the top of a 12-meter-tall tree. A snake is approaching its nest at the base of the pillar from a distance of 30 metres from the bottom of the tree. When the parrot sees the snake, it pounces on it. How far away from the nest is the snake captured if their speeds are equal?

- a. 17.4 m      b. 15 m  
c. 14.8 m      d. 13.2 m

**42.** Harry and Mark were about to take part in an interschool science experiment competition that his school had

organised. They had some experiments going on. As seen in the picture below, Mark was holding a 12 cm rod that he was holding between a flashlight and a wall. By noting that the rod is 45 cm from the wall and 15 cm from the light, Harry was able to determine the length of the shadow cast on the wall. What is the shadow's length?

- a. 75 cm                      b. 48 cm  
c. 24 cm                      d. 20 cm

43. Paul has arranged for her daughter's destination wedding reception. He considered setting up tents for guests rather than rooms to stay in order to make a wedding unique. He expects 150 guests at his wedding reception. He intends to build a conical tent with 4 m<sup>2</sup> of ground space and 20 m<sup>3</sup> of breathing space. What should the conical tent's height be?

- a. 8 m                          b. 10 m  
c. 12 m                      d. 15 m

44. Princeton School of Management has held an entrance exam for four schools for grade 10 students who want to take the next step by majoring in mathematics. They received a favourable response. They recorded the number of students who took part in an entrance in the table below, but they forgot to record the number of students who took part in school III. Determine the number of candidates who took the test from school III if the average score of candidates from all four schools is 66.

Schools	No. of Candidates	Average Score
I.	60	75
II.	48	80
III.	Not available	55
IV.	40	50

- a. 55                              b. 53  
c. 52                              d. 50

45. Sia has a bag containing 12 yellow and red coloured balls, x of which are yellow. She chose one ball at random. She put 6 more yellow balls in the bag, so the probability of drawing a yellow ball is now double that of the previous probability of drawing a yellow ball. What is the value of x?

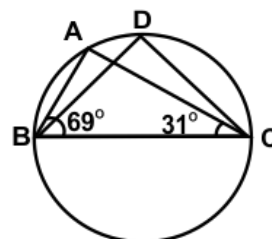
- a. 3                                b. 4  
c. 5                                d. 6

46. Andrea, Max, Milo, Federick, and Changs were tossing coins. Toss a coin 200 times so that five coins are simultaneously tossed 1000 times, and the number of heads observed and recorded at each toss. The number of tosses with 0, 1, 2, 3, 4, and 5 heads was recorded in the table below. Determine the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

- a. 2                                b. 2.47  
c. 3                                d. 3.5

47. In figure,  $\angle ABC = 69^\circ$ ,  $\angle ACB = 31^\circ$ , find  $\angle BDC$ .



- a.  $51^\circ$                       b.  $80^\circ$   
c.  $88^\circ$                         d.  $90^\circ$

**48.** What are the maximum marks if a student needs to achieve 33% of the total marks to pass, and they scored 125 marks but failed by 40 marks?

- a. 300                              b. 400  
c. 500                              d. 600

**49.** Find two numbers such that five times the greater exceeds four times the lesser by 22 and three times the

greater together with seven times the lesser is 32.

- a. 6,2                              b. 4,3  
c. 3,5                              d. 1,3

**50.** If you take a number, double it, add 20 to the result, and that equals the same value as if you were to multiply the number by 8 and subtract 4 from the product, what is the value of the original number?

- a. 4                                      b. 5  
c. 6                                      d. 7

## Answer Key

- |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (a)  | 3. (b)  | 4. (a)  | 5. (c)  | 6. (b)  | 7. (b)  |
| 8. (a)  | 9. (a)  | 10. (d) | 11. (b) | 12. (b) | 13. (c) | 14. (d) |
| 15. (a) | 16. (c) | 17. (b) | 18. (c) | 19. (a) | 20. (a) | 21. (c) |
| 22. (c) | 23. (d) | 24. (a) | 25. (c) | 26. (a) | 27. (b) | 28. (a) |
| 29. (a) | 30. (a) | 31. (d) | 32. (a) | 33. (c) | 34. (b) | 35. (b) |
| 36. (c) | 37. (b) | 38. (b) | 39. (a) | 40. (b) | 41. (a) | 42. (b) |
| 43. (d) | 44. (c) | 45. (a) | 46. (b) | 47. (b) | 48. (c) | 49. (a) |
| 50. (a) |         |         |         |         |         |         |