Grade 10


## CREST Mathematics Olympiad (CMO) Sample Paper

## Pattern and Marking Scheme

| Grade | Topic/Section | No. of <br> Questions | Marks per <br> Question | Total <br> Marks |
| :---: | :---: | :---: | :---: | :---: |
| Grade 10 | Practical Mathematics | 40 | 1 | 40 |
|  | Achiever's Section | 10 | 2 | 20 |
| Grand Total |  | $\mathbf{5 0}$ |  | $\mathbf{6 0}$ |

The total duration of the exam is 60 minutes.

## Syllabus

Section 1: Real Numbers, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Arithmetic Progressions, Triangles, Coordinate Geometry, Introduction to Trigonometry, Some Applications of Trigonometry, Circles, Constructions, Areas Related to Circles, Surface Areas and Volumes, Statistics, Probability.

Achievers Section: Higher Order Thinking Questions - Syllabus as per Section 1
For more details, visit https://www.crestolympiads.com/maths-olympiad-cmo

## Practical Mathematics (Each Question is 1 Mark)

1. If $3 y+4 x=1, y=x+5$ and $5 y+b x=3$ are concurrent, find the value of ' $b$ '.
a. 1
b. 3
c. 6
d. 0
2. Two vertices of a triangle are $(5,-1)$ and $(-2,3)$. If the orthocentre of the triangle is the origin, find the third vertex.
a. $(4,7)$
b. $(4,-7)$
c. $(-4,7)$
d. $(-4,-7)$
3. For an acute angle $\theta, \sin \theta+\cos \theta$ takes the greatest value when $\theta$ is:
a. $30^{\circ}$
b. $45^{\circ}$
c. $60^{\circ}$
d. $90^{\circ}$
4. If the coefficients of $r^{\text {th }}$ term and $(r+1)^{\text {th }}$ term in the expansion of $(1+x)^{20}$ are in the ratio $1: 2$, what is the value of $r$ ?
a. 6
b. 7
c. 8
d. 9
5. Which term is numerically the greatest term in the expansion of $(3+2 x)^{49}$, when $x=1 / 5$ ?
a. $4^{\text {th }}$ term
b. $5^{\text {th }}$ term
c. $6^{\text {th }}$ term
d. $7^{\text {th }}$ term
6. If the sum of the roots of the equation $a x^{2}+b x+c=0$ is equal to the sum of their squares, then which one of the following is correct?
a. $a^{2}+b^{2}=c^{2}$
b. $a^{2}+b^{2}=a+b$
c. $2 a c=a b+b^{2}$
d. $2 c+b=0$
7. If $\cos x /(1+\operatorname{cosec} x)+\cos x /(\operatorname{cosec} x-1)=2$, then which one of the following is one of the values of $x$ ?
a. $\pi / 2$
b. $\pi / 3$
c. $\pi / 4$
d. $\pi / 6$
8. If $p_{1}$ and $p_{2}$ are two odd prime numbers such that $p_{1}>p_{2}$, then $p_{1}{ }^{2}-p_{2}{ }^{2}$ is:
a. An even number
b. An odd number
c. An odd prime number
d. A prime number
9. If the first, second and last terms of an AP are $a, b$ and $c$, respectively, then the sum is:
a. $[(a+b)(a+c-2 b)] /[2(b-a)]$
b. $[(b+c)(a+b-2 c)][2(b-a)]$
c. $[(a+c)(b+c-2 a)] /[2(b-a)]$
d. None of these
10. The areas of two similar triangles are $81 \mathrm{~cm}^{2}$ and $49 \mathrm{~cm}^{2}$, respectively, then what will be the ratio of their corresponding medians?
a. $7: 9$
b. $9: 81$
c. $9: 7$
d. $81: 7$
11. To divide a line segment $A B$ in the ratio 2: 5, first a ray $A X$ is drawn, so that $\angle B A X$ is an acute angle and then at an equal distance points are marked on the ray $A X$ such that the minimum number of this point is:
a. 2
b. 5
c. 4
d. 7
12. Write the general term in the expansion of $\left(x^{2}-y\right)^{6}$ :
a. $(-1)^{r} .{ }^{6} \mathrm{C}_{r} \mathrm{x}^{12-2 r}$
b. $(-1)^{r}{ }^{6} \mathrm{C}_{r} \mathrm{x}^{12-2 r} \cdot \mathrm{y}^{r}$
c. $(1)^{r}{ }^{6} C_{r} \cdot y^{r}$
d. $(1)^{r}{ }^{6} \mathrm{C}_{\mathrm{r}}{ }^{12} \cdot \mathrm{y}^{r}$
13. If $\alpha, \beta$ are the roots of the equation $x^{2}-2 x+3=0$, then find the equation whose roots are $1 / \alpha^{2}$ and $1 / \beta^{2}$.
a. $x^{2}+2 x+1=0$
b. $9 x^{2}+2 x+1=0$
c. $9 x^{2}-2 x+1=0$
d. $9 x^{2}+2 x-1=0$
14. The houses of a row are numbered consecutively from 1 to 49 . If there is a value of $x$ such that the sum of the numbers of the houses preceding the house numbered $x$ is equal to the sum of the numbers of the houses following it. Find the value of $x$.
a. $x=33$
b. $x=43$
c. $x=39$
d. $x=35$
15. Find the values of $a$ and $b$ for which $3 x^{3}-a x^{2}-74 x+b$ is a multiple of $x^{2}+2 x-24$.
a. $\mathrm{a}=-5, \mathrm{~b}=24$
b. $a=5, b=24$
c. $a=13, b=16$
d. $a=-13, b=16$
16. What is the value of the expression $\left[(a-b)^{3}+(b-c)^{3}+(c-a)^{3}\right] /[(a-b)(b-c)(c-a)]$ ?
a. 1
b. 0
c. 2
d. 3
17. A bag contains 63 cards (numbered 1, 2, 3, .....,63). Two cards are picked at random from the bag (one after another and without replacement). What is the probability that the sum of the numbers of both the cards drawn is even?
a. $11 / 21$
b. $34 / 63$
c. $7 / 11$
d. None of the above
18. The average weight of $A, B$ and $C$ is 84 kg . If $D$ joins the group, the average weight of the group becomes 80 kg . If another man $E$ who weighs 3 kg more than D replaces A , then the average of $B, C, D$ and $E$ becomes 79 kg . What is the weight of $A$ ?
a. 64 kg
b. 72 kg
c. 75 kg
d. 80 kg
19. The following steps are involved in finding a number, if the positive number is less than its square by 30 . Arrange them in sequential order:
(A) $x^{2}-x-30=0$
(B) $x=6$
(C) $x^{2}-x=30$
(D) $(x+5)(x-6)=0$
a. CADB
b. DCBA
c. DCAB
d. CDAB
20. A certain strain of virus occurs three times every 25 minutes. In how much time will it become 729 times its initial value?
a. 125 minutes
b. 100 minutes
c. 150 minutes
d. 175 minutes
21. If $1 /(a+b), 1 /(b+c)$, and $1 /(c+a)$ are in AP, then which of the following is true?
a. $a^{2}, b^{2}$, and $c^{2}$ are in AP.
b. $a^{2}, b^{2}$, and $c^{2}$ are in GP.
c. $b^{2}, a^{2}$ and $c^{2}$ are in HP.
d. $b^{2}, a^{2}$ and $c^{2}$ are in AP.
22. How many 5 -digit odd numbers can be formed using the digits $2,3,5,7,8$, and 9 if every digit can occur at most once in any number?
a. 400
b. 475
c. 420
d. 480
23. A square is drawn by joining midpoints of the sides of a square. Another square is drawn inside the second square in the same way and the process is continued indefinitely. If, the side of the first square is 16 cm , then what is the sum of the areas of all the squares?
a. $256 \mathrm{~cm}^{2}$
b. $512 \mathrm{~cm}^{2}$
c. $1024 \mathrm{~cm}^{2}$
d. $512 / 3 \mathrm{~cm}^{2}$
24. The shadow of a pole standing on a horizontal plane is $d$ metre longer when the Sun's altitude is $\alpha$ than when it is $\beta$. What is the height of the pole?
a. $d \cos \alpha \cos \beta / \cos (\alpha-\beta)$
b. $d \sin \alpha \cos \beta / \sin (\alpha-\beta)$
c. $d \sin \alpha \sin \beta / \sin (\beta-\alpha)$
d. $d \sin \beta \cos \alpha / \cos (\alpha+\beta)$
25. The average mark obtained by the students in a class is 43 . If the average marks obtained by 25 boys are 40 and the average marks obtained by the girl students are 48 , then what is the number of girl students in the class?
a. 15
b. 17
c. 18
d. 20
26. The arithmetic mean of the squares of the first n natural number is:
a. $n(n+1)(2 n+1) / 6$
b. $n(n+1)(2 n+1) / 2$
c. $(n+1)(2 n+1) / 6$
d. $(n+1)(2 n+1) / 3$
27. What is the difference between a plant and an electric heater in terms of the energy they are associated with?
a. 0
b. $a+b+c$
c. 1
d. $a b c$
28. Find the value of $x+y$ in the solution of the equations $x / 4+y / 3=5 / 12$ and $x / 2+y=1$ :
a. $1 / 2$
b. $3 / 2$
c. 2
d. $5 / 2$
29. The volume of a pyramid whose base is an equilateral triangle is $12 \mathrm{~cm}^{3}$. If the height of the pyramid is $3 \sqrt{ } 3 \mathrm{~cm}$, then find the length of each side of the base:
a. 2 cm
b. 3 cm
c. 4 cm
d. 6 cm
30. In how many other ways can the alphabet of the word DECIDE be arranged?
a. $6!/ 2!$
b. $6!/ 3!$
c. $6!/(2!\times 2!)$
d. $[6!/(2!* 2!)]-1$
$31.30 \%$ of the items were sold at a profit of $40 \%$ while the remaining were sold at $x \%$ loss. If the overall loss is $10 \%$, find the value of x .
a. $30 \%$
b. $30.42 \%$
c. $31.42 \%$
d. $31.24 \%$
31. Simplify the following expression: $\left[(a-b)^{3}-(a+b)^{3}\right] / 2+a\left(a^{2}+3 b^{2}\right)$
a. $a^{3}-b^{3}$
b. $(a+b)^{3}$
c. $a^{2}+b^{3}$
d. $(a-b)^{3}$
32. A three-digit number was chosen at random. Find the probability that its hundred's digit, ten's digit and unit's digit are consecutive integers in descending order.
a. 1/75
b. $4 / 225$
c. $2 / 225$
d. $1 / 45$
33. In $\triangle A B C, \angle B=90^{\circ}$. $P, Q$ and $R$ are the midpoints of $A B, B C$ and $A C$, respectively. Which of the following is true?
a. A, P, Q and R
b. B, P, R and Q
c. $C, Q P$ and $R$
d. All of these
34. If $\tan A=1 / 2$ and $\tan B=1 / 3$, then which of the following is true?
a. $\mathrm{A}+\mathrm{B}=\pi^{\mathrm{c}} / 4$
b. $\mathrm{A}-\mathrm{B}=\pi^{\mathrm{c}} / 4$
c. $2(A+B)=\pi^{c} / 4$
d. $\mathrm{AB}=\pi^{\mathrm{c} / 4}$
35. If $\sin A=\sqrt{3} / 2$ and $A$ is an acute angle, then find the value of $(\tan A-\cot A) /(\sqrt{3}+\operatorname{cosec} A)$ :
a. $-2 / 5$
b. $2 / 5$
c. $2 /(3+2 \sqrt{ } 3)$
d. -2
36. Ken and Paul can complete a job in 40 days and 50 days, respectively. They worked on alternative days to complete it. Find the minimum possible time in which they could have completed it:
a. $44 \frac{2}{5}$ days
b. $44 \frac{1}{2}$ days
c. $443 / 5$ days
d. $444 / 5$ days
37. In the adjoining figure, the bottom of the glass has a hemispherical raised portion. If the glass is filled with orange juice, then find the quantity of juice which a person will get:

a. $135 \mathrm{~m} \mathrm{~cm}^{3}$
b. $117 \mathrm{mcm}^{3}$
c. $99 \pi \mathrm{~cm}^{3}$
d. $36 \pi \mathrm{~cm}^{3}$
38. In the given figure, PQRS is a square of side $7 \sqrt{ } 2 \mathrm{~cm}$. With $P$ and $R$ as centres and $P Q$ as radius, the arcs QAS and QBS are drawn, respectively. Find the area of the shaded region (in $\mathrm{cm}^{2}$ ).

a. 50
b. 48
c. 44
d. 42
39. In the given figure, $A B\left|\mid D E\right.$ and the area of the parallelogram $A B F D$ is $24 \mathrm{~cm}^{2}$. Find the areas of triangles AFB, AGB, and AEB.

a. $8 \mathrm{~cm}^{2}$
b. $12 \mathrm{~cm}^{2}$
c. $10 \mathrm{~cm}^{2}$
d. $14 \mathrm{~cm}^{2}$

## Achiever's Section (Each Question is 2 Marks)

41. Find the quadratic equation whose roots are reciprocal of the roots of the equation $3 x^{2}-20 x+$ $17=0$.
a. $17 x^{2}-20 x+3=0$
b. $17 x^{2}+20 x+3=0$
c. $17 x^{2}-20 x-3=0$
d. $17 x^{2}+20 x-3=0$
42. Two regular polygons are such that the ratio between their number of sides is $1: 2$ and the ratio of measures of their interior angles is $3: 4$. Find the number of sides of each polygon.
a. 5,10
b. 6,12
c. 4,8
d. 2,3
43. $f(x)=x^{4}-2 x^{3}+3 x^{2}-a x+b$ is a polynomial such that when it is divided by $(x-1)$ and $(x+1)$, the remainders are 5 and 19, respectively. Determine the remainder when $f(x)$ is divided by ( $x$ $-2)$.
a. 6
b. 10
c. 2
d. 8
44. Perpendiculars are drawn from the vertex of the obtuse angles of a rhombus to its sides. The length of each perpendicular is equal to a unit. The distance between their feet is equal to $b$ units. Find the area of the rhombus.
a. $\sqrt{ }\left(a^{2}+b^{2}\right) / 2 \sqrt{ }\left(b^{2}-a^{2}\right)$
b. $2 a b / 2 \sqrt{ }\left(b^{2}-a^{2}\right)$
c. $a b^{2} / 2 \sqrt{ }\left(b^{2}-a^{2}\right)$
d. $2 a^{2} b^{2} / 2 \sqrt{ }\left(b^{2}-a^{2}\right)$
45. Inside a triangular park, there is a flower bed forming a similar triangle. Around the flower bed runs a uniform path of such a width that the sides of the park are exactly double the corresponding sides of the flower bed. Find the ratio of the area of the path to the flower bed:
a. $1: 1$
b. 1:2
c. $1: 3$
d. $3: 1$
46. The angles of elevation of the top of a tower from two points at distances $m$ and $n$ metres are complementary. If the two points and the base of the tower are on the same straight line, then what will be the height of the tower?
a. $\sqrt{ }(m n)$
b. mn
c. $m / n$
d. $n / m$
47. In the binomial expansion of $(a-b)^{n}, n \geq 5$ the sum of the $5^{\text {th }}$ and $6^{\text {th }}$ terms is zero. Find the value of $a / b$.
a. $(\mathrm{n}-5) / 6$
b. $(n-4) / 5$
c. $5 /(n-4)$
d. $6 /(n-5)$
48. A tower stands vertically on the ground. From a point on the ground which is 30 m away from the foot of a tower, the angle of elevation of the top of the tower is found to be $45^{\circ}$. Find the height of the tower.
a. 28 m
b. 30 m
c. 32 m
d. 35 m
49. Which among the following is a singleton set?
a. The set A consisting of all prime numbers less than 10 .
b. The set B consisting of all squares up to 10 .
c. The set C consisting of all numbers up to 10 that are neither prime nor composite.
d. The set $D$ of all odd numbers up to 10 .
50. W borrowed a certain sum of money from $X$ at the rate of $10 \%$ per annum under simple interest and lent one-fourth of the amount to Y at $8 \%$ per annum under simple interest and the remaining amount to $Z$ at $15 \%$ per annum under simple interest. If at the end of 15 years, W made a profit of $\$ 5850$ in the deal, then find the sum that W had lent to Z :
a. $\$ 24,500$
b. $\$ 12,000$
c. $\$ 9,000$
d. $\$ 18,600$

## Answer Key

| 1. | c | 2. | d | 3. | b | 4. | b | 5. | c | 6. | c | 7. | c |  |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8. | a | 9. | c | 10. | c | 11. | d | 12. | b | 13. | b | 14. | d |  |  |  |  |  |  |  |  |  |  |
| 15. | a | 16. | d | 17. | d | 18. | c | 19. | a | 20. | c | 21. | a |  |  |  |  |  |  |  |  |  |  |
| 22. | d | 23. | b | 24. | c | 25. | a | 26. | c | 27. | b | 28. | b |  |  |  |  |  |  |  |  |  |  |
| 29. | c | 30. | d | 31. | c | 32. | d | 33. | c | 34. | b | 35. | a |  |  |  |  |  |  |  |  |  |  |
| 36. | b | 37. | a | 38. | b | 39. | d | 40. | b | 41. | a | 42. | a |  |  |  |  |  |  |  |  |  |  |
| 43. | b | 44. | c | 45. | d | 46. | a | 47. | b | 48. | b | 49. | c |  |  |  |  |  |  |  |  |  |  |
| 50. | c |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

