MATHEMATICS WORKBOOK

9

For the preparation of National
& International Olympiads

- Chapter-wise practice exercises
- Previous year paper

CREST Mathematics Olympiad (CMO)

Mathematics Olympiad

Exams Preparation Book

CMO | IMO | UMO | iOM | UIMO | HMO

Grade 9

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CREST Mathematics Olympiad Workbook for Grade 9

Second Edition

Disclaimer: The information in the Workbook is to give you the path to success but it does not guarantee 100% success as the strategy is completely dependent on its execution. And it is based on previous year papers of CMO exam.

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Preface

We are pleased to launch a thoroughly revised edition of this workbook. We welcome feedback from students, teachers, educators and parents. For improvements in the next edition, please send your suggestions at info@crestolympiads.com.

CREST Olympiads is one of the largest Olympiad Exams with students from more than 25 countries. The objective of these exams is to build competitive spirit while evaluating students on conceptual understanding of the concepts.

We strive to provide a superior learning experience, and this workbook is designed to complement the school studies and prepare the students for various competitive exams including the CREST Olympiads. This workbook provides a crisp summary of the topics followed by the practice questions. These questions encourage the students to think analytically, to be creative and to come up with solutions of their own. There's a previous year paper given at the end of this workbook for the students to attempt after completing the syllabus. This paper should be attempted in 1 hour to get an assessment of the student's preparation for the final exam.

Publishers

Chapter 1

Number System

Numbers are a core part of mathematics. In this workbook, the students will be introduced to several topics related to numbers and their applications.

Natural Number

Natural numbers are a part of the number system which includes all the positive integers from 1 to infinity.
For example, 74 is a natural number.

Whole Number

Natural numbers including zero form the set of whole numbers.
For example: 0, 5, 14, 854, etc. are whole numbers.

Integers

Counting numbers, zero and negatives of counting numbers form the set of integers.
For example, 21, 4, 0, and -2048 are integers, while 9.75, +5, and $\sqrt{2}$ are not integers.

Rational Numbers

A number 'r' is called a rational number if it can be written in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0. For example, 24 can be considered as 'r'. It can be represented as 48 by 2 where p = 48 and q = 2 which is not equal to 0.

Irrational Numbers

Any number that cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers and q ≠ 0, is an irrational number. Examples: $\sqrt{2}$, 1.010024563…, e, π.

Equivalent Rational Number

The rational number whose numerator and denominator both are equal, or they are reducible to equal.

Example:

$$\frac{1}{2} = \frac{2}{4} = \frac{10}{20} = \frac{25}{50} = \frac{47}{94}$$

Real Numbers

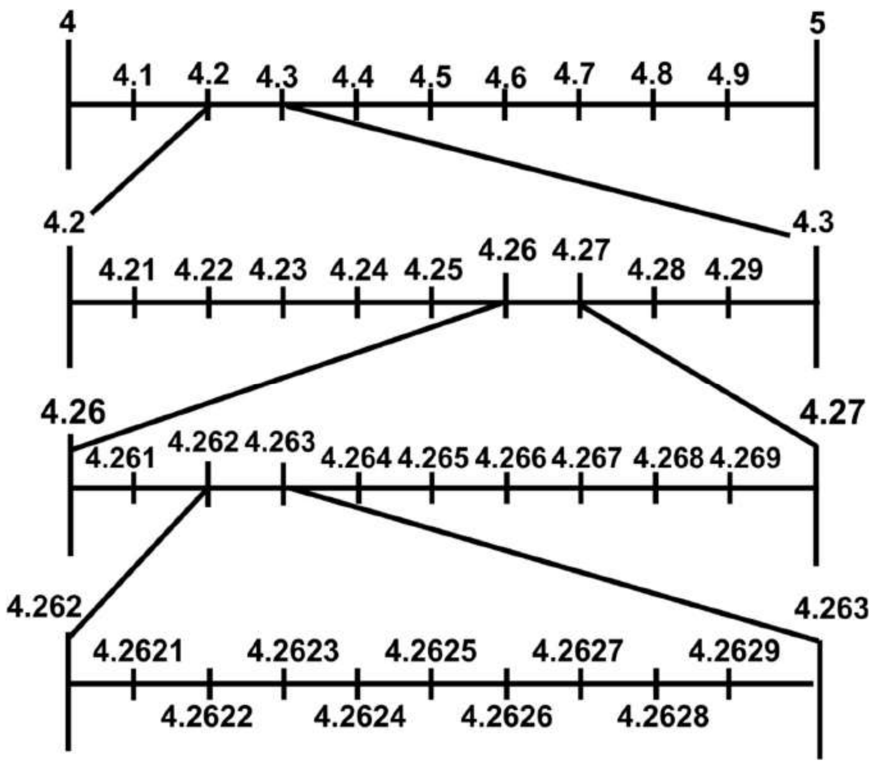
Any number which can be represented on the number line is a Real Number (R). It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

Representing Real Numbers on the Number Line

To represent the real numbers on the number line we use the process of successive magnification in which we visualise the numbers through a magnifying glass on the number line.
Example:

Mark $4.\overline{26}$ on the number line up to 4 decimal places.

Step 1: The number lies between 4 and 5, so we divide it into 10 equal parts. Now for the first decimal place, we will mark the number between 4.2 and 4.3.
Step 2: Now we will divide it into 10 equal parts again. The second decimal place will be between 4.26 and 4.27.
Step 3: Now we will again divide it into 10 equal parts. The third decimal place will be between 4.262 and 4.263.
Step 4: By doing the same process again we will mark the point at 4.2626.



Identities for Irrational Numbers

Arithmetic operations between:

- Rational and irrational will give an irrational number.
- Irrational and irrational will give a rational or irrational number.

If a and b are real numbers, then:

- $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
- $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$
- $(\sqrt{a} + \sqrt{b})$
- $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$
- $(\sqrt{a} + \sqrt{b})(\sqrt{c} - \sqrt{d}) = \sqrt{ac} - \sqrt{ad} + \sqrt{bc} - \sqrt{bd}$
- $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$

Laws of Exponents for Real Numbers

If a, b, m, and n are real numbers then:

- $a^m \times a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $a^m b^m = (ab)^m$

Here, a and b are the bases, and m, and n are exponents.

Exponential Representation of Irrational Numbers

Let a > 0 be a real number and p and q be rational numbers, then:

- $a^p \times a^q = a^{p+q}$
- $(a^p)^q = a^{pq}$
- $\frac{a^p}{a^q} = a^{p-q}$
- $a^p b^p = (ab)^p$

Terminating Decimal

A terminating decimal is usually defined as a decimal number that contains a finite number of digits after the decimal point. All terminating decimals are rational numbers that can be written as reduced fractions with denominators containing no prime number factors other than two or five.

Recurring Decimal

A repeating decimal or recurring decimal is a decimal representation of a number whose digits are periodic, and the infinitely repeated portion is not zero. It can be shown that a number is rational if and only if its decimal representation is repeating or terminating.

Example 1: Compare the surds $A = \sqrt{10} + \sqrt{9}$ and $B = \sqrt{13} + \sqrt{6}$:

a. Surd A < Surd B
b. Surd A = Surd B
c. Surd A > Surd B
d. None of these

Solution 1: c

Since there is a positive sign, by squaring both the surds, we get,

$$A^2 = \left(\sqrt{10} + \sqrt{9}\right)^2 = 10 + 9 + 2\sqrt{90}$$

$$= 19 + 2\sqrt{90}$$

$$B^2 = \left(\sqrt{13} + \sqrt{6}\right)^2 = \left(\sqrt{13} + \sqrt{6}\right)^2$$

$$= 13 + 6 + 2\sqrt{78}$$

$$= 19 + 2\sqrt{78}$$

As, 90 > 78

Surd A > Surd B

Example 2: Simplify:

$$(3 + \sqrt{7})(5 + \sqrt{11})$$

a. $15 + 5\sqrt{7} + 3\sqrt{11} + \sqrt{77}$
b. $15 + 5\sqrt{7} + 3\sqrt{11} + \sqrt{77}$
c. $15 + 5\sqrt{7} + 3\sqrt{11} + \sqrt{77}$
d. $15 + 5\sqrt{7} + 3\sqrt{11} + \sqrt{77}$

Solution 2:

We will use the identity

$$(\sqrt{p} + \sqrt{q})(\sqrt{r} + \sqrt{s}) = \sqrt{pr} + \sqrt{ps} + \sqrt{qr} + \sqrt{qs}$$

$$(3 + \sqrt{7})(5 - \sqrt{11}) = 15 + 5\sqrt{7} + 3\sqrt{11} + \sqrt{77}$$

Example 3: Rationalise the numerator of $\frac{4 - \sqrt{6+x}}{x-10}$.

a. $\frac{1}{(3 + \sqrt{6+x})}$
b. $\frac{-1}{(4 + \sqrt{6+x})}$
c. $\frac{-1}{(3 + \sqrt{6+x})}$
d. $\frac{1}{(4 + \sqrt{3+x})}$

Solution 3: b

Rationalizing factor of $4 - \sqrt{6+x}$ is $4 + \sqrt{6+x}$.

$$\frac{4 - \sqrt{6+x}}{x - 10} = \frac{4 - \sqrt{6+x}}{x - 10} \times \frac{4 + \sqrt{6+x}}{4 + \sqrt{6+x}}$$

$$= \frac{16 - (6 + x)}{(x - 10)(4 + \sqrt{6 + x})}$$

$$= \frac{10 - x}{(x - 10)(4 + \sqrt{6 + x})}$$

$$= \frac{-1}{(4 + \sqrt{6 + x})}$$

Example 4: Simplify the following by rationalising the denominator:

$$\frac{3\sqrt{5} - \sqrt{7}}{3\sqrt{3} + \sqrt{2}}$$

a. $\frac{(9\sqrt{15} + 3\sqrt{10} - 3\sqrt{21} + \sqrt{14})}{25}$

b. $\frac{(9\sqrt{15} - 3\sqrt{10} + 3\sqrt{21} + \sqrt{14})}{25}$

c. $\frac{(9\sqrt{15} - 3\sqrt{10} - 3\sqrt{21} + \sqrt{14})}{25}$

d. $\frac{(9\sqrt{15} + 3\sqrt{10} + 3\sqrt{21} + \sqrt{14})}{25}$

Solution 4: c

$$\frac{3\sqrt{5} - \sqrt{7}}{3\sqrt{3} + \sqrt{2}} = \frac{(3\sqrt{5} - \sqrt{7})(3\sqrt{3} - \sqrt{2})}{(3\sqrt{3} + \sqrt{2})(3\sqrt{3} - \sqrt{2}}$$

$$= \frac{(9\sqrt{15} - 3\sqrt{10} - 3\sqrt{21} + \sqrt{14})}{\left[(3\sqrt{3})^2 - (\sqrt{2})^2\right]}$$

[Since $x^2 - y^2 = (x + y)(x - y)$]

$$= \frac{(9\sqrt{15} - 3\sqrt{10} - 3\sqrt{21} + \sqrt{14})}{(27-2)}$$

$$= \frac{(9\sqrt{15} - 3\sqrt{10} - 3\sqrt{21} + \sqrt{14})}{25}$$

Practice Questions

1. If $\frac{a}{\sqrt{8}+\sqrt{14}+\sqrt{12}+\sqrt{21}} = \frac{\sqrt{8}-\sqrt{14}-\sqrt{12}+\sqrt{21}}{k}$ then a =?

a. $\frac{6}{K}$
b. $\frac{3}{K}$
c. $\frac{10}{K}$
d. $\frac{12}{K}$

2. If $x = \sqrt[3]{2+\sqrt{3}}$, find the value of $x^3 + \frac{1}{x^3}$.

a. 2
b. 4
c. 8
d. 9

3. Find the simplest rationalising factor of $7^{\frac{1}{3}} + 7^{-\frac{1}{3}}$.

a. $\left(7^{\frac{2}{3}}\right) - 1 + \left(7^{-\frac{2}{3}}\right)$
b. $\left(7^{\frac{1}{3}}\right) - \left(7^{-\frac{1}{3}}\right)$
c. $\left(7^{\frac{2}{3}}\right) + 1 + \left(7^{-\frac{2}{3}}\right)$
d. $\left(7^{\frac{2}{3}}\right) + 1 - \left(7^{-\frac{2}{3}}\right)$

4. If a = $\sqrt{2}$, b = $\sqrt[3]{3}$, and c = $\sqrt[4]{4}$, then which of the following relation holds?

a. a < b < c
b. a = c < b
c. a < c < b
d. b = a < c

5. $\sqrt{324+2\sqrt{323}} - \sqrt{324-2\sqrt{323}} = ?$

a. 2
b. 1
c. 36
d. $2\sqrt{323}$

6. The smallest of $\sqrt[3]{7}, \sqrt[4]{5}, \sqrt[4]{9}, \sqrt[3]{8}$ is:

a. $\sqrt[3]{7}$
b. $\sqrt[4]{5}$
c. $\sqrt[4]{9}$
d. $\sqrt[3]{8}$

7. $\frac{3}{\sqrt{3}+1} + \frac{3}{\sqrt{5}+\sqrt{3}} + \frac{3}{\sqrt{7}+\sqrt{5}} + \cdots \ldots \ldots + \frac{3}{\sqrt{25}+\sqrt{23}} = ?$

a. $\sqrt{23}$
b. $3\sqrt{23}$
c. 3
d. 6

8. If $\frac{\left(l+\frac{1}{m}\right)^{l}\left(l-\frac{1}{m}\right)^{m}}{\left(m+\frac{1}{l}\right)^{l}\left(m-\frac{1}{l}\right)^{m}} = \left(\frac{l}{m}\right)^{x}$, then $x =$

a. $l - m$
b. $l + m$
c. $m - l$
d. lm

9. A = $\sqrt{8} + \sqrt{7}$, B = $\sqrt{10} + \sqrt{5}$ and C = $\sqrt{12} + \sqrt{3}$ then:

a. C > B > A
b. B > A > C
c. A > B > C
d. C > A > B

10. Simplify $\frac{b^{\frac{1}{2}} + b^{-\frac{1}{2}}}{1 - b} + \frac{1 - b^{-\frac{1}{2}}}{1 + \sqrt{b}}$

a. 1
b. 0
c. $\frac{2}{(1 - b)}$
d. $1 + b$

11. Find the last two digits of 3^{2021}.

a. 63
b. 21
c. 3
d. 17

12. Find the sum of all the digits of the result of the subtraction 100^{99} - 999.

a. 874
b. 1874
c. 1756
d. 1755

13. Find the value of:

$$\left(\sqrt[6]{64} - \sqrt{7\frac{1}{9}}\right)^{2}$$

a. $\frac{4}{3}$
b. $\frac{2}{3}$
c. $\frac{-2}{3}$
d. $\frac{4}{9}$

14. If $5^{x} - 5^{x-1} = 20$, what is the value of $(2x)^{x}$?

a. 2
b. 16
c. 4
d. 5

15. If m = $\frac{1}{2-\sqrt{3}}$, then find the value of $m^3 - 2m^2 - 7m + 10$.

a. 14
b. 4
c. 8
d. 16

16. If a = $7 - 4\sqrt{3}$, then $\frac{a^2+1}{7a}$ = ?

a. 2
b. 1
c. 7
d. 3

17. The value of $\left(\sqrt[6]{23 - 4\sqrt{33}}\right)\left(\sqrt[3]{\sqrt{11} + 2\sqrt{3}}\right)$ is equal to:

a. 0
b. $\sqrt{3}$
c. 1
d. $\sqrt[6]{3}$

18. The value of $\frac{2}{\sqrt{12-2\sqrt{35}}}$ is equal to:

a. $\frac{1}{2}(\sqrt{7}+\sqrt{5})$
b. $\sqrt{7}+\sqrt{5}$
c. $\frac{1}{2}(\sqrt{7}-\sqrt{5})$
d. $\sqrt{7}-\sqrt{5}$

19. Which of the following surd is the smallest?

$\sqrt{16}-\sqrt{14}, \sqrt{5}-\sqrt{3}, \sqrt{7}-\sqrt{5}, \sqrt{19}-\sqrt{17}$

a. $\sqrt{16}-\sqrt{14}$
b. $\sqrt{5}-\sqrt{3}$
c. $\sqrt{7}-\sqrt{5}$
d. $\sqrt{19}-\sqrt{17}$

20. $\sqrt{17\sqrt{17\sqrt{17\sqrt{17 \ldots\ldots\ldots.11\ terms}}}}$ = ?

a. $\sqrt[2048]{17^{11}}$
b. $\sqrt[2048]{17}$
c. $\sqrt[2048]{17^{2047}}$
d. $\sqrt[2048]{17^{2049}}$