MATHEMATICS WORKBOOK

10

For the preparation of National & International Olympiads

- Chapter-wise practice exercises
- Previous year paper

CREST Mathematics Olympiad (CMO)

Mathematics Olympiad

Exams Preparation Book

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Grade 10

#CRESTInnovator

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CREST Mathematics Olympiad Workbook for Grade 10

Fourth Edition

Disclaimer: The information in the Workbook is to give you the path to success but it does not guarantee 100% success as the strategy is completely dependent on its execution. And it is based on previous year papers of CMO exam.

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Preface

We are pleased to launch a thoroughly revised edition of this workbook. We welcome feedback from students, teachers, educators and parents. For improvements in the next edition, please send your suggestions at info@crestolympiads.com. Our team will make an effort to work on those suggestions. The status of the improvements can be checked at https://www.crestolympiads.com/corrections-class10-652

CREST Olympiads is one of the largest Olympiad Exams with students from more than 60 countries. The objective of these exams is to build a competitive spirit while evaluating students on conceptual understanding of the concepts.

We strive to provide a superior learning experience, and this workbook is designed to complement the school studies and prepare the students for various competitive exams including the CREST Olympiads. This workbook provides a crisp summary of the topics followed by the practice questions. These questions encourage the students to think analytically, to be creative and to come up with solutions of their own. There is a previous year's paper given at the end of this workbook for the students to attempt after completing the syllabus. This paper should be attempted in 1 hour to get an assessment of the student's preparation for the final exam.

Publishers

Chapter 1

Number System and Polynomials

Number System

A number system is defined as a system of writing to express numbers.

Natural Numbers

Natural numbers are a part of the number system which includes all the positive integers from 1 to infinity.
For example, 74 is a natural number.

Whole Number

Whole numbers include all natural numbers and 0
They are real numbers that do not include fractions, decimals, and negative integers.
For example: 0, 5,14, 854, etc. are whole numbers.

Integers

An integer is a number that can be written without a fractional component.
For example, 21, 4, 0, and −2048 are integers, while 9.75, 5+, and $\sqrt{2}$ are not.
The set of integers consists of zero, the positive natural numbers, also called whole numbers or counting numbers, and their additive inverses.

Rational Numbers

A number 'r' is called a rational number if it can be written in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.
For example, 24 can be considered as 'r'. It can be represented as 48 by 2 where p = 48 and q = 2 which is not equal to 0.

Irrational Numbers

Any number that cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers and q ≠ 0, is an irrational number.
Examples: $\sqrt{2}$, 1.010024563…, e, π

Real Numbers

Any number which can be represented on the number line is a Real Number (R). It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

Properties of Irrational Numbers

1. Addition, subtraction, multiplication, and division of two irrational number is may or may not be irrational.
2. Addition of rational and irrational number is always irrational.
3. Subtraction of rational and irrational number is always irrational.
4. Multiplication of a non-zero rational and irrational is always irrational.
5. Division of a non-zero rational and irrational is always irrational.

If a and b are real numbers, then:

- $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a - b$
- $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$
- $(\sqrt{a}+\sqrt{b})$
- $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ad}+\sqrt{bc}+\sqrt{bd}$
- $(\sqrt{a}+\sqrt{b})(\sqrt{c}-\sqrt{d}) = \sqrt{ac}-\sqrt{ad}+\sqrt{bc}-\sqrt{bd}$
- $(\sqrt{a}+\sqrt{b})^2 = a + 2\sqrt{ab} + b$

Laws of Exponents for Real Numbers

If a, b, m and n are real numbers then:

- $a^m \times a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $a^m b^m = (ab)^m$

Here, a and b are the bases, and m and n are exponents.

Exponential Representation of Irrational Numbers

Let a > 0 be a real number and p and q be rational numbers, then:

- $a^p \times a^q = a^{p+q}$
- $(a^p)^q = a^{pq}$
- $\frac{a^p}{a^q} = a^{p-q}$
- $a^p b^p = (ab)^p$

Terminating Decimal

A terminating decimal is usually defined as a decimal number that contains a finite number of digits after the decimal point.
All terminating decimals are rational numbers that can be written as reduced fractions with denominators containing no prime number factors other than two or five.

Recurring Decimal

A repeating decimal or recurring decimal is decimal representation of a number whose digits are periodic (repeating its values at regular intervals) and the infinitely repeated portion is not zero.

Polynomials

Polynomials are expressions with one or more terms with a non-zero coefficient. A polynomial can have more than one term. An algebraic expression $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_n$ is a polynomial where $a_0, a_1, \ldots\ldots\ldots a_n$ are real numbers and n is non-negative integer.

- An expression is called a rational expression if it can be written in the form p(x) / q(x) where p(x) and q(x) are polynomials and q(x) ≠ 0.
- Every polynomial is a rational expression, but every rational expression need not be a polynomial.
- The power of a variable in a polynomial must be a whole number.
- A polynomial d(x) is called a divisor of a polynomial p(x) if p(x) = d(x) q(x) for some polynomial q(x).

Term

In the polynomial, each expression in it is called a **term.**
For example, if $4x^2 + 9x + 7$ is a polynomial then $4x^2$, 9x and 7 are the terms of the polynomial.

Coefficient

Each term of a polynomial has a coefficient.
Suppose if 5x + 1 is a polynomial then the coefficient of x is 5.

Monomial, Binomial and Trinomial

Polynomials of one term, two terms and three terms are called monomial, binomial, and trinomial respectively.

Examples of monomial: 5, 3x, $2x^2$
Examples of binomial: 2z + 3, 3x - 1
Examples of trinomial: $4x^2+9x+7$

Degree of a Polynomial

The highest power of the polynomial is called the degree of the polynomial.

Linear Polynomial

A polynomial of degree one is called a linear polynomial.
Example: $3x + 1$

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.
Example: $2x^2 + 1x + 1$

Cubic Polynomial

A polynomial of degree three is called a cubic polynomial.
Example: $y^3 + 8$

Biquadratic polynomial

A polynomial of degree four is called a biquadratic polynomial.
Example: $x^4 + x^3 - x^2 + x + 1$

Note

- A real number 'a' is a zero of a polynomial p(x) if p(a) = 0. 'a' is also called the root of the equation p(x) = 0.
- Every linear polynomial in one variable has a unique zero.
- A non-zero constant polynomial has no zero.
- Every real number is a zero of the zero polynomial.
- The degree of a non-zero constant polynomial is zero.
- The degree of a zero polynomial is not defined.

If p(x) and g(x) are two polynomials such that degree of p(x) ≥ degree of g(x) and g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that p(x) = g(x) q(x) + r(x).

Zeroes of a Polynomial

A zero of a polynomial p(x) is the value of x for which the value of p(x) is 0. If k is a zero of p(x), then p(k) = 0.
For example, consider a polynomial $p(x) = x^2 - 3x + 2$.
When x = 1, the value of p(x) will be equal to
p (1) = 12 - 3 × 1 + 2
= 1 - 3 + 2 = 0
Since p(x) = 0 at x = 1, we say that 1 is a zero of the polynomial $x^2 - 3x + 2$.

Factor Theorem

Let f(x) be a polynomial of degree $n \geq 1$ and 'a' be any real number. Then

- (x - a) is a factor of f(x) if f(a) = 0.
- f(a) = 0 if (x - a) is a factor of f(x).

If x - 1 is a factor of a polynomial of degree 'n' then the sum of its coefficients is zero.

Remainder Theorem

If p(x) is any polynomial of degree greater than or equal to 1 and p(x) is divided by the linear polynomial x - a (where 'a' is any real number) then the remainder is p(a).
We can express p(x) as p(x) = (x - a) q(x) + r(x) where q(x) is the quotient and r(x) is the remainder.
The process of writing an algebraic expression as the product of two or more algebraic expressions is called factorization.

Some Important Identities

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + b^3 + 3a^2b + 3ab^2$
6. $(a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - b^3 - 3a^2b + 3ab^2$
7. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
8. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
9. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
10. If a + b + c = 0 then $a^3 + b^3 + c^3 = 3abc$.
11. $(x + a)(x + b) = x^2 + (a + b)x + ab$

Example 1: In how many ways can 480 be resolved into two factors?

a. 18
b. 12
c. 24
d. 36

Solution 1: b

$480 = 2^5 \times 3^1 \times 5^1$
Number of factors of 480 = (5 + 1) (1 + 1) (1 + 1) = 24.
Number of ways in which 480 can be resolved into two factors = $\frac{24}{2}$ = 12

Example 2: Express the rational expression $\frac{x^2-2x-3}{2x^2-3x-5}$ in its lowest terms.

a. $\frac{(x+3)}{(2x-5)}$
b. $\frac{(x-3)}{(2x+5)}$.
c. $\frac{(x+3)}{(2x+5)}$
d. $\frac{(x-3)}{(2x-5)}$

Solution 2: d

Factorising both the numerator and the denominator of the given expression, we have
$\frac{x^2-2x-3}{2x^2-3x-5} = \frac{(x+1)(x-3)}{(2x-5)(x+1)}$
HCF of the numerator and denominator is x + 1.
Dividing both numerator and denominator of the given expression by (x + 1), we have $\frac{(x-3)}{(2x-5)}$ which is in its lowest terms.

Example 3: During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

a. 4 packs of pencils, 3 packs of crayons
b. 3 packs of pencils, 4 packs of crayons
c. 3 packs of pencils, 3 packs of crayons
d. 4 packs of pencils, 4 packs of crayons

Solution 3: a

From given data
LCM of 24 and 32
32 = 2 × 2 × 2 × 2 × 2
24 = 2 × 2 × 2 × 3

LCM (32, 24) = 2 × 2 × 2 × 2 × 2 ×3 = 96
24 pencils = 1 pack
96 pencils = 4 × 24 pencils = 4 packs
32 crayons = 1 pack
96 crayons = 3 × 32 crayons = 3 packs
If want pencils and crayons the same in number, we have to buy 4 packs of pencils and 3 packs of crayons.

Example 4: What must be subtracted from the polynomial f(x) = $x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$?

a. (x - 2)
b. (2x - 3)
c. (x + 3)
d. (2x + 3)

Solution 4: b

$$
\begin{array}{r|l}
 & x^2 + 6x + 8 \\
\hline
x^2 - 4x + 3 & x^4 + 2x^3 - 13x^2 - 12x + 21 \\
 & \pm x^4 \mp 4x^3 \pm 3x^2 \\
\hline
 & \quad 6x^3 - 16x^2 - 12x + 21 \\
 & \quad \pm 6x^3 \mp 24x^2 \pm 18x \\
\hline
 & \qquad 8x^2 - 30x + 21 \\
 & \qquad \pm 8x^2 \mp 32x \pm 24 \\
\hline
 & \qquad\qquad 2x - 3
\end{array}
$$

(2x - 3) should be subtracted from $x^4 + 2x^3 - 13x^2 - 12x + 21$

Example 5: Solve for x, if $\frac{\log 225}{\log 15} = \log x$.

a. 10
b. 100
c. 20
d. 200

Solution 5: b

$\log x = \frac{\log 225}{\log 15}$

$\log x = \frac{\log(15 \times 15)}{\log 15}$

$\log x = \frac{\log(15^2)}{\log 15}$

$\log x = \frac{2 \log 15}{\log 15}$

$\log x = 2$

Or

$\log_{10} x = 2$

$10^2 = x$

$x = 10 \times 10$

$x = 100$

Practice Questions

1. If A = $\frac{x+1}{2x-1}$, B = $\frac{2x+1}{3x+2}$ and C = $\frac{4x-5}{2x^2+5x-3}$ then find 4A - 3B + C.

a. $\frac{44x^2-94x+5}{6x^2+19x^2+x-6}$

b. $\frac{44x^2+94x+5}{6x^2+19x^2-x-6}$

c. $\frac{44x^2+94x+5}{6x^2+19x^2+x-6}$

d. $\frac{44x^2+94x+5}{6x^2-19x^2+x-6}$

2. If the zeroes of the rational expression (3x + 2a) (2x + 1) are $-\frac{1}{2}$ and $\frac{b}{3}$ then the value of a is:

a. $-2b$

b. $-\frac{b}{2}$

c. $-\frac{b}{3}$

d. None of these

3. A shopkeeper wants to use a minimum number of boxes at the same time all the boxes should contain the same no. of pencils. But each box must contain only one type of pencil. How many pencils per box should he pack if he wants to pack 612 orange pencils and 342 red pencils?

a. 18

b. 21

c. 20

d. 16

4. Graham and Mary call each other at intervals of 12 minutes and 18 minutes respectively. If they both got busy tone at 10 pm, After how many minutes do they get busy tone again, at the earliest?

a. 26

b. 16

c. 36

d. 18

5. There are 78 boys and 45 girls in a class. These students are arranged in rows grouping in the Athletics event, each row consists of only either boys or girls, and every row contains an equal number of students. Find the minimum number of rows in which all the students can be arranged.

a. 43

b. 45

c. 41

d. 51

6. Louis, Harvey, Mike and Pearson start running around a circular track simultaneously. If they complete one round in 15, 10, 12 and 18 minutes respectively, after how much time will they next meet at the starting point? Choose the right answer from the given options.

a. 160

b. 200

c. 190

d. 180

7. Simplify $\frac{a^2-(b-c)^2}{(a+c)^2-b^2}+\frac{b^2-(a-c)^2}{(a+b)^2-c^2}+\frac{c^2-(a-b)^2}{(b+c)^2-a^2}$

a. 0
b. 1
c. $a + b + c$
d. $\frac{1}{a+b+c}$

8. Albie was given two polynomials P and Q. She used long division and found G.C.D of P and Q to be 3x –2, and the first two quotients obtained were x + 2 and 2x + 1. Find P and Q. (The degree of P > the degree of Q).

a. $p(x) = 6x^3 + 11x^2 + x + 6$, $q(x) = 6x^2 + x + 2$
b. $p(x) = 6x^3 + 11x^2 - x + 6$, $q(x) = 6x^2 - x + 2$
c. $p(x) = 6x^3 - 11x^2 + x - 6$, $q(x) = 6x^2 - x - 2$
d. $p(x) = 6x^3 + 11x^2 - x - 6$, $q(x) = 6x^2 - x - 2$

9. Given $ax^2 + bx + c$ is a quadratic polynomial in x and leaves remainders 7, 12 and 19, respectively, when divided by (x + 1), (x + 2) and (x + 3). Find the value of a + b + c.

a. 3
b. 4
c. 2
d. 1

10. Resolve $\frac{1}{(x-4)(x^2+3)}$ into a partial fraction.

a. $\frac{1}{19}\left[\frac{1}{x-4}+\frac{x+4}{x^2+3}\right]$
b. $\frac{1}{19}\left[\frac{1}{x-4}-\frac{x+3}{x^2+3}\right]$
c. $\frac{1}{19}\left[\frac{1}{x-4}-\frac{x+4}{x^2+3}\right]$
d. $\frac{1}{19}\left[\frac{1}{x-4}-\frac{x+3}{x^2+3}\right]$

11. Find the sum of $\frac{1}{\sqrt{9}+\sqrt{10}}+\frac{1}{\sqrt{10}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{12}}+\ldots$.. up to 112 terms.

a. 19
b. 8
c. 1
d. 2

12. If $(x) = \log\left(\frac{1+x}{1-x}\right)$, find $f\left(\frac{2x}{1+x^2}\right)$ in terms of f(x).

a. 0 f(x)
b. 2 f(x)
c. 3 f(x)
d. f(x)

13. If $\frac{4x^2+5x+6}{x^2(x+3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}$, then,

a. 16
b. 26
c. 36
d. 46

14. Find the middle terms in the expansion of $(3x - 5y)^7$.

a. ${}^7C_3\,(3x)^4(5y)^3$
b. ${}^7C_4\,(3x)^3(5y)^4$
c. ${}^7C_3\,(3x)^3(5y)^4$
d. Both (a) and (b)

15. If $a = \log_{26}13$, $b = \log_{39}26$, $c = \log_{52}39$, then find 1 + abc.

a. bc
b. 2ca
c. 2bc
d. ab

16. Find x, if $\log_{5x}\sqrt{x} + \log_{5\sqrt{x}}x = 0$.

a. $5^{-\frac{6}{5}}$
b. 2
c. 1
d. Both (A) and (C)

17. Find the term independent of 'x' in the expansion of $(1 + x^2)\left(1 + \frac{1}{x^2}\right)^4$.

a. 60
b. 70
c. 45
d. 75

18. If $x = 1 + \log_a bc, y = 1 + \log_b ca, z = 1 + \log_c ab$, find xyz.

a. xy + yz + zx
b. x + y + z
c. 2(x + y + z)
d. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

19. If $\log_2 x \cdot \log_3 x \cdot \log_5 x = \log_2 x \cdot \log_3 x + \log_3 x \cdot \log_5 x + \log_5 x \cdot \log_2 x$ find x.

a. 0
b. 1
c. 15
d. 30

20. Find the sum of the coefficients of the terms of the expansion $(1 + 3x + 7x^2)^9$.

a. 9^9
b. 22^9
c. 11^9
d. 4^9